

Multi-Objective Optimization Framework for Emergency Building Sweep Operations: A Mixed-Integer Programming Approach with Uncertainty

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Abstract. Emergency evacuation efficiency directly impacts survival rates during fire incidents. Statistics indicate that survival probability decreases by 7–10% per minute after eight minutes without rescue. We develop a multi-objective route optimization framework balancing sweep completion time and occupant waiting duration. The baseline Mixed Integer Linear Programming (MILP) model minimizes weighted makespan and waiting costs across homogeneous settings. For complex scenarios involving heterogeneous occupants, we integrate a Fuzzy Membership Degree model handling type uncertainty. The framework extends to multi-floor buildings with responder volume optimization, validated through progressive scenarios from simple office layouts to complex college buildings. An Ant Colony Optimization algorithm combined with local search efficiently solves large-scale instances. Extensions incorporate dynamic injury deterioration in fire emergencies and drone-assisted inspection. Sensitivity analysis confirms model robustness, with makespan remaining stable across weight variations. Results demonstrate that layout complexity significantly impacts required personnel, with responder needs increasing from 2 to 13 as building complexity and dynamic effects intensify.

Keywords: Multiple Traveling Salesman Problem; Mixed Integer Linear Programming; Multi-Objective Optimization; Emergency Evacuation; Ant Colony Optimization.

1. Introduction

During fire emergencies and hazardous incidents, responders conduct systematic room-by-room sweeps to ensure complete building evacuation. The efficiency of these operations proves critical, as delays or missed areas lead to casualties. Empirical data from the Firefighter Rescue Survey reveals that victims located within eight *minutes* have a 66% survival rate, decreasing by 7–10% per minute thereafter [1,2]. This temporal sensitivity necessitates mathematical frameworks optimizing responder routing strategies.

The challenge intensifies in large or multi-floor buildings where complex layouts and limited crew coordination cause *significant* delays (Figure 1). While existing evacuation models often focus on occupant egress, the responder routing problem—determining optimal sweep paths that minimize both completion time *and* occupant exposure risk—remains underexplored. This study addresses this gap by formulating the building sweep as a Multi-Objective Multiple Traveling Salesman Problem (mTSP), balancing efficiency against safety.

Our contributions include: (1) a baseline MILP model incorporating operational constraints and dual objectives; (2) fuzzy set theory integration for handling occupant type uncertainty; (3) multi-floor *extensions* with *responder* volume optimization; (4) hybrid metaheuristic algorithms for large-scale instances; and (5) realistic extensions modeling dynamic injury deterioration and technology-assisted operations.



(a) Firefighters guiding evacuation.

(b) Fire suppression operations.

Figure 1: Emergency response operations requiring optimized sweep strategies. Images illustrate the dual objectives of rapid evacuation coordination and fire control.

2. Problem Formulation

2.1. Problem Definition

A building evacuation reaches “all-clear” status when: (i) all occupants exit rooms and evacuate the building; (ii) responders complete verification procedures; and (iii) responders exit through the nearest exit. We consider two types of evacuation: building sweeps, where external hazards require verification, but occupants are uninjured, and fire rescues, where internal fires cause potential injuries deteriorating with time.

Buildings comprise rooms with varying areas and functions (offices, apartments, hotel rooms, classrooms), affecting inspection time per square meter. Occupants are heterogeneous—adults (A), children (C), and elderly (E) each requiring different inspection durations. Responders are trained firefighters capable of rapid movement, systematic search in complex terrain, and orderly evacuation coordination.

Operational procedures require responders to: (i) enter, inspect, and evacuate each room before continuing; (ii) use stairs exclusively for floor transitions; and (iii) mark cleared rooms to prevent redundant coverage [3].

2.2. Key Assumptions

Assumption 1. Responders are homogeneous in speed and processing efficiency, reflecting standardized training protocols [4].

Assumption 2. Responders move in straight lines between locations. In open corridors, straight paths represent optimal movement; for complex layouts, obstacle effects are captured in room inspection times.

Assumption 3. Responders thoroughly inspect entire rooms following standard operating procedures.

Assumption 4. Complete room status information is available through IoT sensors, with instantaneous responder communication.

Assumption 5. Emergency types are predetermined based on initial sensor data, allowing appropriate model activation.

3. Baseline MILP Model

3.1. Model Formulation

Consider a building with room set $R = \{1, 2, \dots, N\}$ and responder set $F = \{1, 2, \dots, M\}$. Let $V = R \cup \{\text{Exit}\}$ denote all *nodes*. Parameters include: n_r^A, n_r^C, n_r^E (adult, child, elderly counts in room r); $P_r = n_r^A + n_r^C + n_r^E$ (total occupants); A_r (room area); β_r (inspection time per square meter); v_f (responder speed); v_{evac} (occupant evacuation speed); T_A, T_C, T_E (per-person inspection times by type); and d_r^{int} (internal distance to door).

Decision *variables* are:

$$x_r^f = \begin{cases} 1 & \text{if room } r \text{ assigned to responder } f \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$y_{uv}^f = \begin{cases} 1 & \text{if responder } f \text{ travels from } u \text{ to } v \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$a_r \geq 0 \quad (\text{arrival time at room } r) \quad (3)$$

$$t_r^{\text{dep}} \geq 0 \quad (\text{departure time from room } r) \quad (4)$$

Room processing involves three phases:

Arrival: $a_r \geq t_v^{\text{dep}} + \text{dist}(v, r)/v_f$ for all predecessor *nodes* v .

Inspection: $S_r = \beta_r A_r + T_A n_r^A + T_C n_r^C + T_E n_r^E$ captures spatial *examination* and occupant verification.

Evacuation: $E_r = p_r \cdot d_r^{\text{int}}/v_{\text{evac}}$ represents coordinated *occupant* egress.

Departure time: $t_r^{\text{dep}} = a_r + S_r + E_r$. Occupant waiting *duration*: $W_r = P_r \cdot a_r + P_r \cdot S_r + E_r$.

Objective Function:

$$\min Z = W_1 \cdot T + W_2 \cdot W_{\text{people}} \quad (5)$$

Where $T = \max_r \{t_r^{\text{dep}} + \min_{v \in \{\text{Exit}\}} \text{dist}(r, v)/v_f\}$, $W_{\text{people}} = \sum_{r \in R} \sum_{f \in F} W_r \cdot x_r^f$, and $W_1 + W_2 = 1$.

Constraints: Each room assigned to exactly one responder ($\sum_{f \in F} x_r^f = 1, \forall r \in R$); route continuity enforced through flow *conservation*; single start/end points per responder.

3.2. Baseline Results

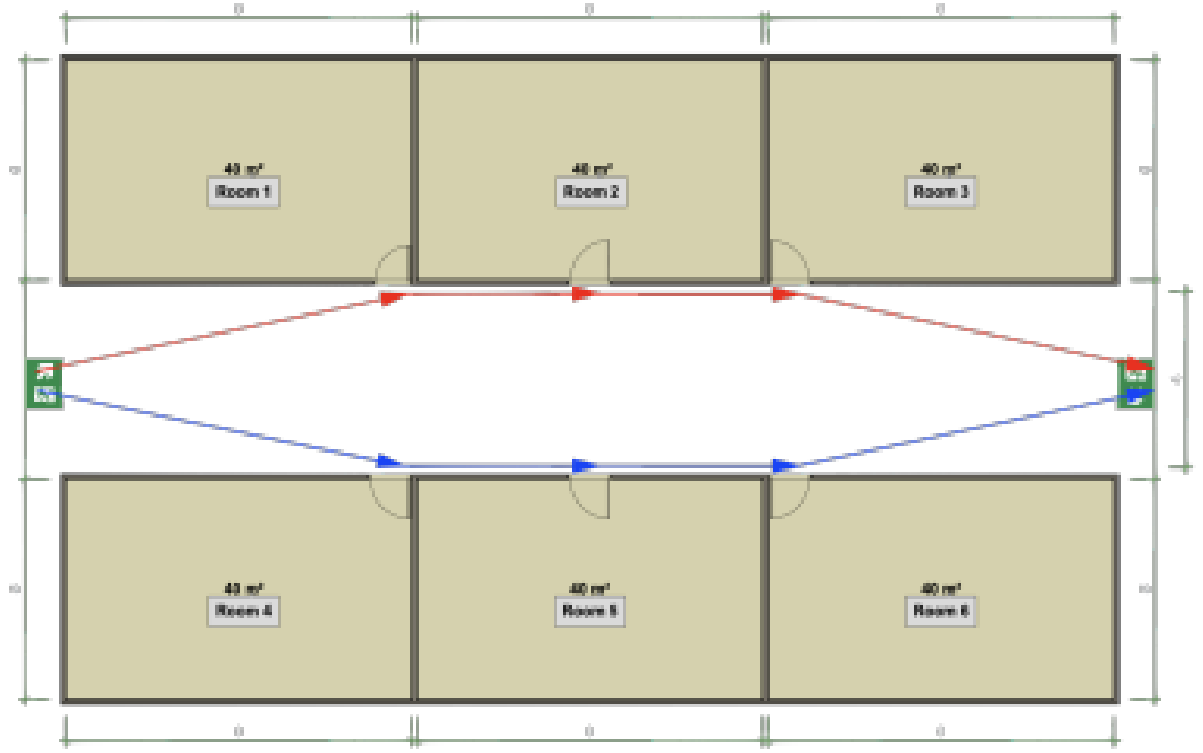


Figure 2: Baseline scenario: Six-room office layout (40 m² rooms, 5 adults each) with optimal routes. Responder 1 follows 1 → 2 → 3; Responder 2 follows 4 → 5 → 6. The symmetric workload distribution achieves balanced completion times.

We test the model on a six-room office layout (Figure 2). Each 40 m² room contains 5 adults. Parameters: $\beta_r = 0.25$ s/m², $v_f = 1.5$ m/s, $v_{\text{evac}} = 0.5$ m/s, $T_A = 5$ s, $W_1 = W_2 = 0.5$. Gurobi Optimizer with 1% optimality gap yields: $Z = 1888.42$, makespan $T = 205.08$ s, average waiting $W_{\text{avg}} = 119.06$ s. The solution demonstrates effective load balancing with symmetric room assignments.

Table 1. Baseline Timing Results for Six-Room Office.

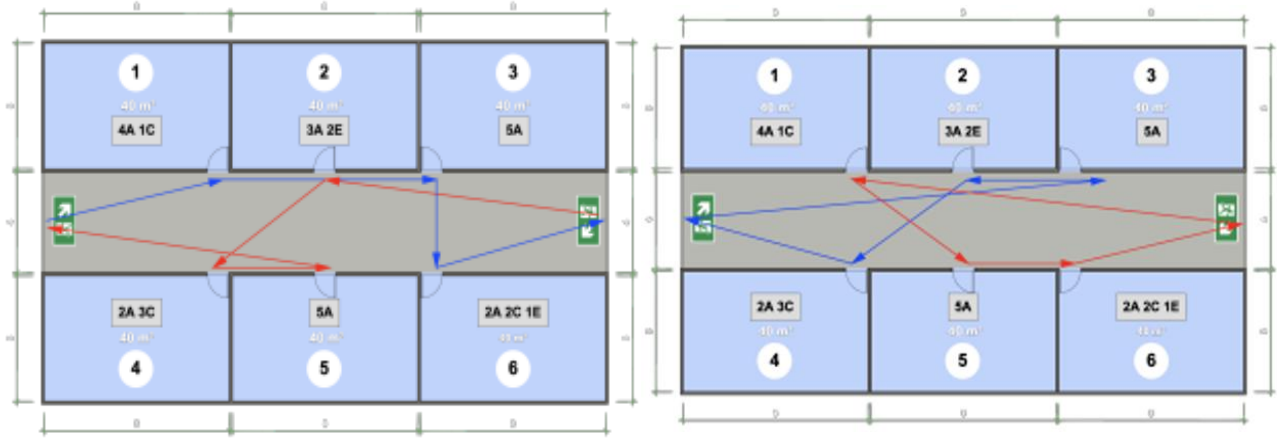
Responder	Room	Arrival (s)	Departure (s)
1	1	2.54	67.54
1	2	69.87	134.87
1	3	137.54	202.54
2	4	2.54	67.54
2	5	69.87	134.87
2	6	137.54	202.54

4. Progressive Scenario Applications

4.1. Scenario 1: Heterogeneous Occupants with Uncertainty

4.1.1. Deterministic Case

We apply the model to a single-floor apartment with six 40 m² rooms containing heterogeneous occupants. Room composition varies: Room 1 (4A, 1C), Room 2 (3A, 2E), Room 3 (5A), Room 4 (2A, 3C), Room 5 (5A), Room 6 (2A, 2C, 1E), where A/C/E denote adults/children/elderly. Updated parameters: $T_C = 10$ s, $T_E = 12$ s reflect increased inspection time for vulnerable populations.



(a) Certainty: $3 \rightarrow 2 \rightarrow 4$ and $1 \rightarrow 5 \rightarrow 6$. (b) Uncertainty: $1 \rightarrow 3 \rightarrow 6$ and $2 \rightarrow 4 \rightarrow 5$.

Figure 3: Scenario 1 optimal routes under certainty (left) and uncertainty (right). Heterogeneity increases makespan by 14.6% versus baseline; uncertainty alters route structure significantly.

Results: $Z = 2099.44$, $T = 235.10$ s, $W_{avg} = 138.79$ s (Figure 3a). Compared to homogeneous baseline ($T = 205.08$ s), heterogeneity increases makespan by 14.6% due to longer inspection times for children and elderly.

4.1.2. Stochastic Case: Fuzzy Membership Model

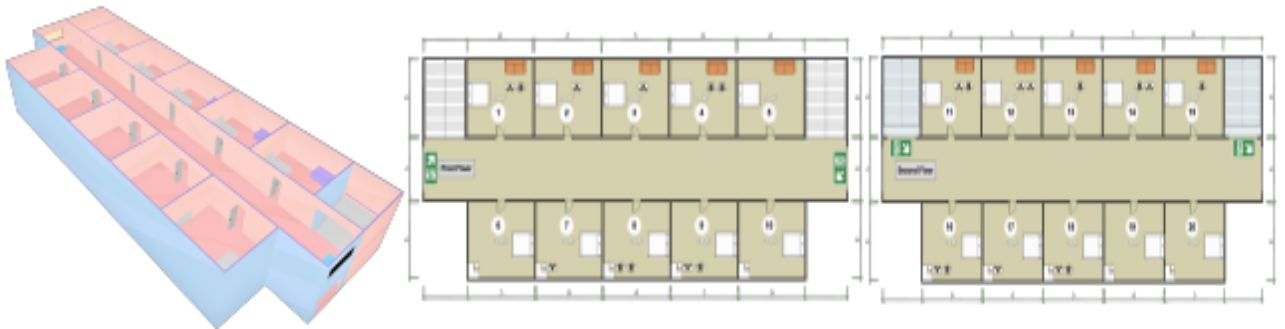
When occupant types are uncertain, we employ fuzzy set theory. For each occupant i in room r , define membership vector $P_i = [P_i^A, P_i^C, P_i^E]$ where P_i^t represents likelihood of type

$t \in \{A, C, E\}$ with $\sum_t P_i^t = 1$. Expected inspection time: $\hat{s}_r = \beta_r A_r + \sum_{i=1}^{p_r} \hat{\tau}_i$ where $\hat{\tau}_i = P_i \cdot [T_A, T_C, T_E]^T$.

With empirically assigned membership degrees, results show: $Z = 1080.24$, $T = 256.12$ s, $W_{avg} = 63.48$ s. Route topology differs from deterministic case (Figure 3b). Best-case realization (all adults): $Z = 823.51$, $T = 207.45$ s. Worst-case (all elderly): $Z = 1365.05$, $T = 417.97$ s. The 65% performance range underscores the value of accurate occupant information.

4.2. Scenario 2: Multi-Floor Hotel

4.2.1. Model Extension



(a) Hotel layout concept

(b) First floor dimensions

(c) Second floor with occupancy

Figure 4: Two-story hotel with double-loaded corridor layout. Rooms are 30 m² with varying occupancy (0–2 adults). Second-floor occupants require supervised stair descent, introducing $D_r = 4.5p_r$ seconds additional time.

We extend to a two-story hotel (Figure 4) with varying occupancy (0, 1, or 2 adults per room). Rooms with zero occupants are excluded via constraint $\varepsilon_{f \in F} X_r^f = 1$ for $p_r > 0$ only. Second-floor

occupants require supervised descent, adding time $D_r = Y \cdot P_r$ where $Y = 4.5$ s per person. Modified departure time: $t_r^{\text{dep}} = a_r + S_r + E_r + D_r$.

4.2.2. Responder Volume Optimization

Fire death data show mortality risk increases sharply beyond 15 minutes [9]. We impose $\max_r W_r \leq 900$ s with minimum team size 2 per OSHA policy [10]:

$$\min n_f \quad \text{subject to} \quad \max W_r \leq 900, \quad n_f \geq 2 \quad (6)$$

For the 20-room hotel, Ant Colony Optimization (ACO) efficiently explores the solution space. Results: $n_f = 2$ suffices with $Z = 2951.51$, $T = 464.23$ s, $W_{\text{avg}} = 226.62$ s (Figure 5). All occupants receive assistance within 900 s threshold.

4.3. Scenario 3: Complex College Building

4.3.1. Large-Scale Framework

We integrate components into a Large-Scale Complex Route Optimization (LSCRO) framework. The test case is an H-shaped, two-story college building with 22 rooms (Figure 6): 16 lecture halls (100 m², 15 occupants) and 6 meeting rooms (60 m², 15 occupants). Manhattan distance replaces Euclidean: $\text{dist}(u, v) = |X_u - X_v| + |Y_u - Y_v|$.

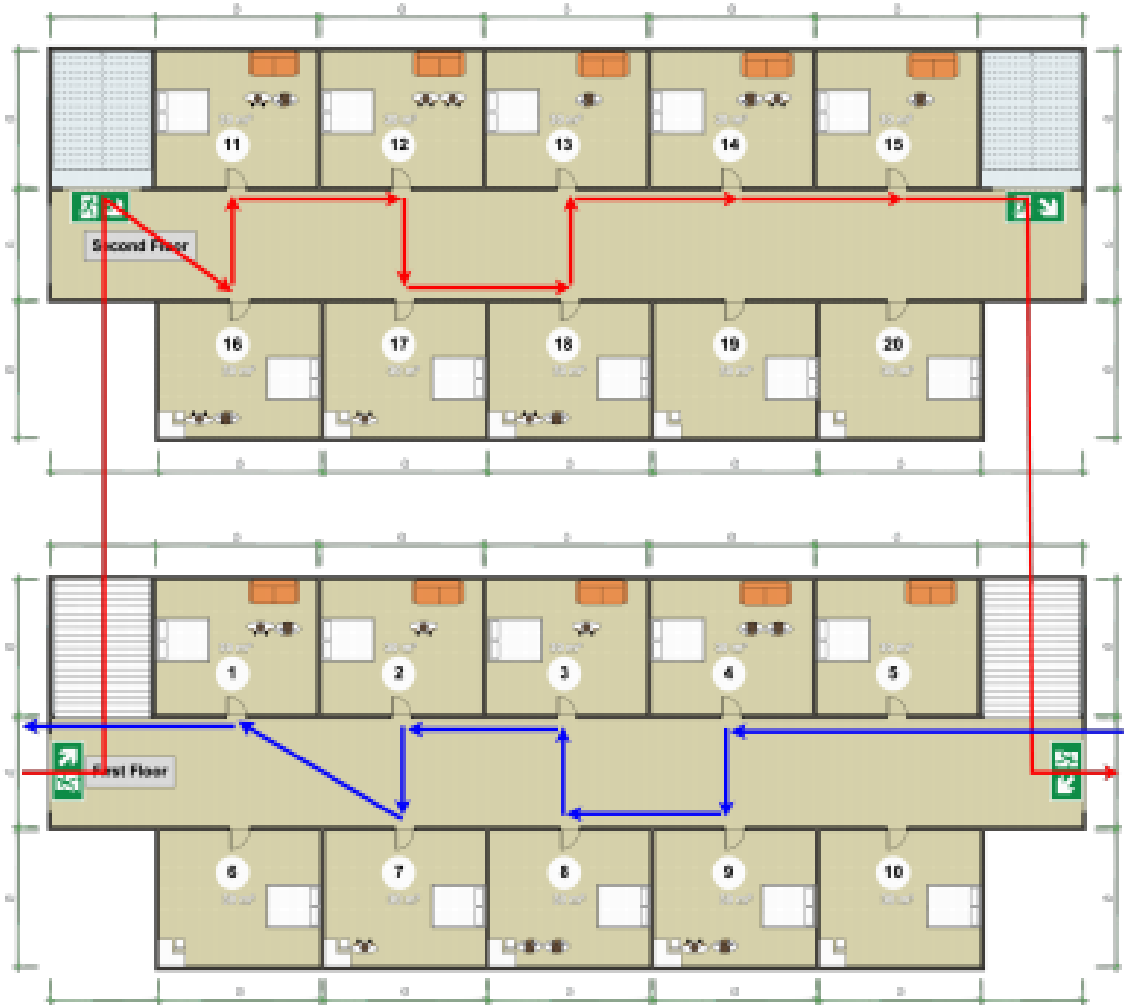
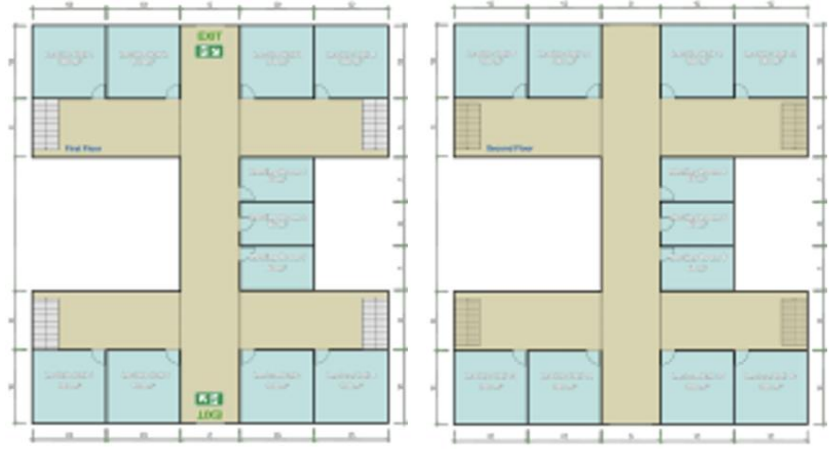


Figure 5: Scenario 2 optimal routes with 2 responders in two-story hotel. Color-coded paths show room assignment across both floors. Responder 1 covers 8 rooms; Responder 2 handles 7 rooms, achieving balanced workload within the 900 s safety threshold.



(a) H-shaped building exterior

(b) Floor layout dimensions (First and Second)

Figure 6: H-shaped two-story college building with 22 rooms. Lecture halls (LH, 100 m²) and meeting rooms (MR, 60 m²) have different capacities and inspection requirements. Manhattan distance models corridor-constrained movement.

4.3.2. In-Class Scenario

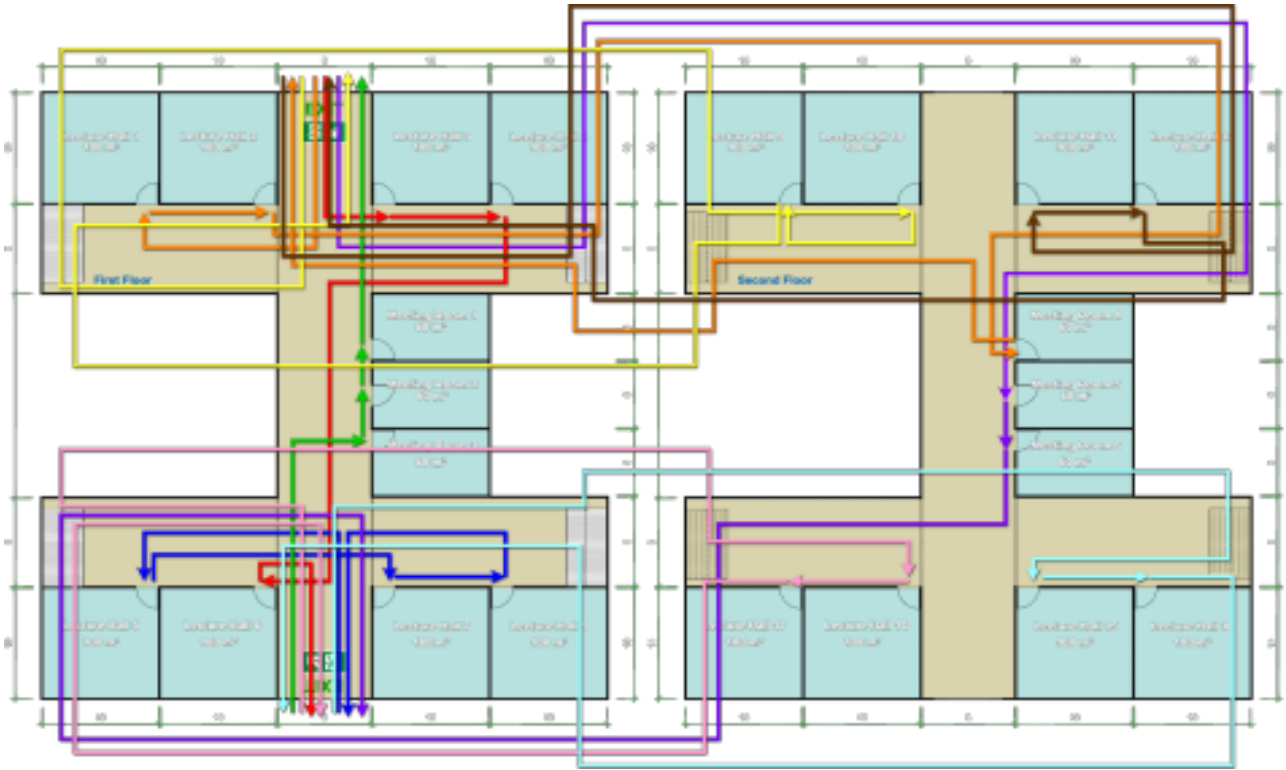


Figure 7: Scenario 3 in-class optimal routes with 9 responders.

Color-coded paths show room assignment and visit sequence. Complex geometry necessitates aggressive personnel deployment to meet 900 s safety threshold ($T = 704.07$ s).

During class hours, occupant composition is known predominantly students not requiring assistance, with some mobility-impaired individuals categorized as "Help" occupants ($T_H = 10$ s vs. $T_N = 5$ s). ACO with 2-opt local search yields: $n_f = 9$ responders necessary, $Z = 143781.79$, $T = 704.07$ s, $W_{avg} = 434.64$ s (Figure 7). Despite high personnel deployment, average waiting nearly doubles versus the simpler hotel, highlighting how architectural complexity dominates staffing requirements.

4.3.3. Out-of-Class Scenario

After hours, occupancy drops to 10 per room with uncertain composition. Applying fuzzy membership with estimated P_r matrices: $n_f = 9$ required, $Z = 40093.07$, $T = 762.73$ s, $W_{avg} =$

361.02 s (Figure 8). Lower occupancy reduces waiting but necessitates identical staffing due to geometric complexity.

5. Model Extensions

5.1. Dynamic Injury Deterioration in Fire Emergencies

Fire rescue differs from building sweeps: injuries worsen with delayed assistance. We model time-dependent evacuation difficulty:

$$E_r = (1 + \sigma \cdot a_r) \cdot p_r \cdot \frac{d_r^{\text{int}}}{v_{\text{evac}}} \quad (7)$$

Where $\sigma = 0.005 \text{ s}^{-1}$ is the deterioration coefficient. The term $(1 + \sigma \cdot a_r)$ captures worsening conditions, incentivizing rapid response.

ACO with 3-opt local search for the college building under fire conditions determines $n_f = 13$ responders required (44% increase over sweep scenario): $Z = 130363.31$, $T = 894.21 \text{ s}$

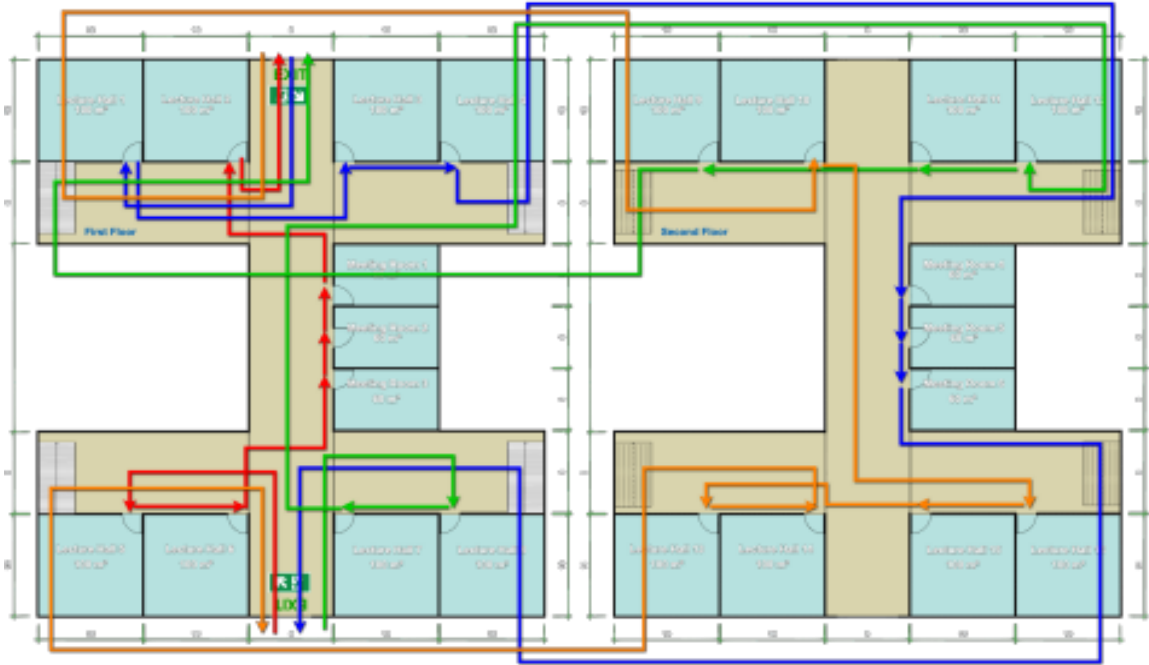


Figure 8. Scenario 3 out-of-class optimal routes with uncertain occupant types.

Fuzzy membership model estimates inspection times. Required personnel remains $n_f = 9$ with $T = 762.73 \text{ s}$, demonstrating that layout topology drives resource allocation.

$W_{\text{avg}} = 393.69 \text{ s}$. Makespan approaches the 900 s threshold, emphasizing that fire emergencies demand parallel deployment.

5.2. Technology-Assisted Operations: Drone Integration

Drones with thermal imaging can rapidly survey buildings, transmitting occupant data [13]. We model a two-stage process: (1) drones inspect ($T_0 = 5 \text{ s}$ per room, $v_d = 3 \text{ m/s}$); (2) firefighters evacuate with perfect information ($s_r = 0$). Combined objective:

$$\min Z = w_1 (T + T_{\text{drone}}) + w_2 \left(W_{\text{people}} + \sum_r p_r \cdot T_{\text{drone}} \right) \quad (8)$$

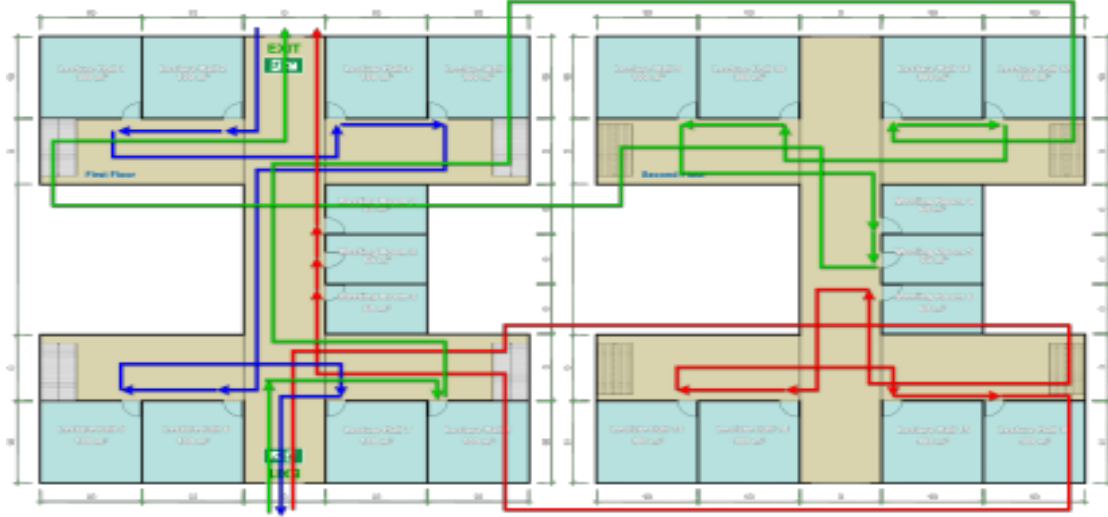


Figure 9: Drone-assisted inspection routes. Three drones complete reconnaissance in $T_{\text{drone}} = 39.57$ s, but sequential operation forces firefighters to idle, paradoxically increasing total cost despite eliminating uncertainty.

With 3 drones and 9 firefighters (Figure 9): $T_{\text{drone}} = 39.57$ s, $T = 887.71$ s, total = 927.27 s, $Z = 152464.73$. Compared to fire rescue without drones ($n_f = 13$, $Z = 130363.31$), drones enable personnel reduction but increase objective by 17% due to enforced sequential operation. Effective human-machine collaboration requires parallel operations.

6. Sensitivity and Robustness Analysis

6.1. Sensitivity to Objective Weights

We examine stability with respect to w_1 , varying from 0.05 to 0.99 using the fire rescue scenario. Table 2 and Figure 10 summarize results.

Table 2. Sensitivity Analysis of Weight Parameter w_1 .

w_1	Z	T (s)	W_{avg} (s)	CPU Time (s)
0.05	247065.62	894.21	748.47	8.79
0.10	234022.61	894.21	705.35	8.74
0.15	220942.46	882.87	663.04	8.65
0.20	207543.47	894.21	622.05	16.53
0.25	195302.10	892.87	582.64	14.17
0.30	182406.93	898.04	541.84	8.69
0.35	169193.49	886.70	500.41	13.59
0.40	155616.92	882.87	458.75	9.28
0.45	143272.36	892.87	420.78	8.80
0.50	130482.28	898.04	379.50	9.18
0.55	117156.00	936.70	340.47	8.63
0.60	104505.43	886.70	307.05	14.20
0.65	91425.34	894.21	275.15	8.65
0.70	78589.92	943.85	234.94	10.88
0.75	65618.41	895.99	194.56	18.76
0.80	52774.79	896.70	155.18	8.73
0.85	39778.48	892.87	114.96	8.91
0.90	26591.04	894.21	74.40	8.89
0.95	13845.97	894.21	39.48	8.83
0.99	3482.15	894.21	9.94	8.58

The objective decreases monotonically as w_1 increases, confirming expected behavior. Critically, makespan exhibits minimal variation (std = 19.7 s, range = [882.87, 943.85]), fluctuating only $\pm 5\%$ despite 20-fold weight changes. This stability indicates robust solution structure—routing topology remains largely invariant. The finding has practical implications: decision-makers can adjust priority between speed and occupant experience without drastically altering deployment strategies.

6.2. Algorithm Robustness

We execute 30 independent ACO runs on standardized hardware (Intel i7, 16GB RAM, Windows 11). Statistical summary:

Table 3. Robustness Analysis: 30 Independent ACO Runs

Metric	Mean	Variance	Min	Max
Execution Time (s)	10.8	10.5	8.95	26.49
Best Objective	130,330	15,800	129,977	130,610
Makespan T (s)	894	21	884.21	931.54

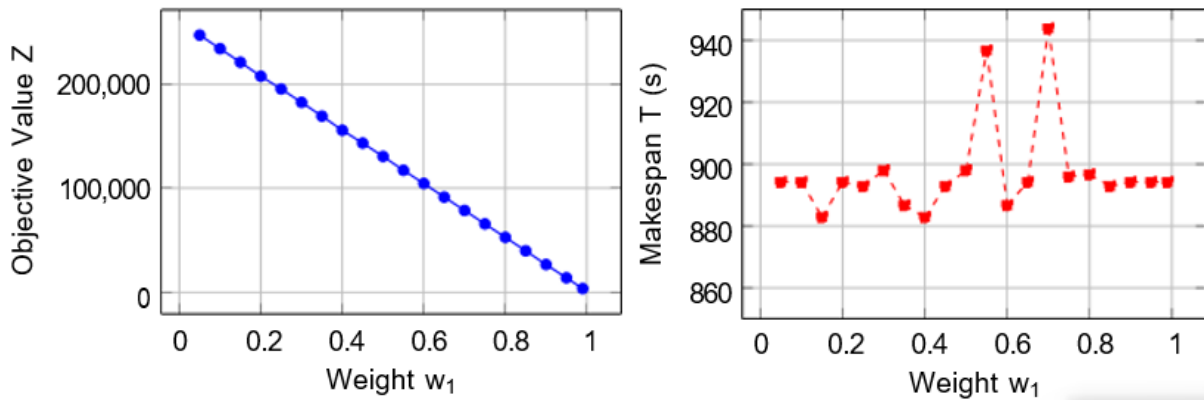


Figure 10: Sensitivity analysis results. Left: Objective value decreases monotonically with w_1 , confirming that higher makespan priority reduces total cost. Right: Makespan remains stable (mean = 901.8 s, std = 19.7 s) across 20-fold weight changes, demonstrating robust solution structure with minimal route topology alterations.

Objective values vary by only 0.49% (coefficient of variation = 0.097%), with makespan exhibiting similar consistency (CV = 0.51%). All runs complete within 27 seconds, validating ACO's reliability for operational use despite inherent stochastic behavior.

7. Discussion

7.1. Key Findings

Layout Complexity Dominates. Personnel requirements increase nonlinearly with building complexity: simple offices (2), hotels (2), college buildings (9), fire emergencies (13). This 6.5-fold increase despite modest occupancy changes (30→390) indicates geometric factors drive resource allocation.

Uncertainty Penalties. Worst-case realizations degrade performance by 65% relative to best-case. Occupancy tracking systems (RFID, IoT) yield measurable routing accuracy improvements.

Dynamic Effects. Fire rescue with deterioration demands 44% more personnel. The coefficient $\sigma = 0.005 \text{ s}^{-1}$ implies 10-minute delays increase evacuation difficulty by 30%.

Technology Integration. Sequential drone-firefighter workflows paradoxically increase costs. Parallel operations require operational redesign and revised command structures.

7.2. Practical Recommendations

Fire departments should: (1) develop building-specific sweep plans using optimization frameworks; (2) conduct regular drills calibrating model parameters; (3) adopt adaptive staffing based on building typology; (4) pilot parallel UAV-ground operations with real-time communication.

8. Conclusion

We developed a comprehensive framework for emergency building sweep optimization, balancing efficiency against safety through multi-objective MILP formulations. Extensions incorporate heterogeneity via fuzzy theory, multi-floor complexity with responder optimization, dynamic deterioration, and drone assistance. Hybrid metaheuristics efficiently solve large-scale instances within seconds. Progressive validation demonstrates scalability; sensitivity analysis confirms robustness. Results quantify the critical importance of layout complexity, uncertainty management, and thoughtful technology integration. The framework provides *actionable* decision support for fire departments, enabling evidence-based staffing and route planning.

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