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A Multi-Process Production Quality Decision Optimization Model Based on Sampling Estimation and Exhaustive Search

Shengtao Yang †, *, Yicheng Sun † and Yile Yang

Shanghai Institute of Technology, Shanghai, China

† These authors also contributed equally to this work

* Corresponding Author Email: shmrktools@163.com

Abstract. This paper proposes a decision-making model that integrates sampling estimation and combinatorial optimization to address quality control and cost optimization in multi-process production systems. First, a sampling estimation method is developed based on hypothesis testing to perform statistical inference on component defect rates, thereby reliably obtaining quality parameters. Building on this foundation, a multi-stage production decision-making model is established, which unifies the inspection and disassembly decisions for components, semi-finished products, and finished goods into discrete variables, and is formulated with the objective of minimizing total cost. For the solution, an exhaustive search algorithm is employed to traverse all decision combinations and obtain the global optimal solution. Additionally, by expanding the defect rate from a deterministic value to a sample estimate, uncertainty is introduced, making the model more closely aligned with real-world production environments. The results indicate that this method can achieve an effective balance among inspection costs, defect handling, and resource recovery, demonstrating good generality and practical value. By integrating statistical inference with discrete optimization methods, this study provides an efficient and feasible modeling and solution approach for quality decision-making in multi-stage production systems.

Keywords: Sampling Estimation; Exhaustive Search; Multi-Process Production.

1. Introduction

In complex manufacturing systems, multi-stage production processes are often characterized by the dynamic coupling of quality uncertainty and cost. How to make efficient decisions under conditions of limited information has become a major research focus in the fields of smart manufacturing and industrial optimization. From a computational perspective, this problem can be abstracted as a multi-stage discrete optimization problem involving uncertain parameters, with the core challenge lying in achieving the coordinated optimization of quality control and cost-benefit trade-offs within a large decision space. Existing methods largely rely on deterministic parameters or local optimization strategies, making it difficult to simultaneously address uncertainty modeling and global optimization. On the one hand, while traditional statistical methods can provide defect rate estimates, they lack deep integration with the decision-making process; on the other hand, classical optimization methods are prone to limitations in search efficiency or getting stuck in suboptimal solutions in high-dimensional discrete spaces [1][2]. To address these issues, this paper proposes a unified algorithmic model framework that integrates sampling estimation with combinatorial optimization. This method uses hypothesis testing to obtain defect rate estimates and embeds them into a multi-stage decision-making model, constructing a high-dimensional binary decision vector to characterize testing and disassembly strategies; furthermore, by combining an exhaustive search mechanism, it achieves global optimization solutions for a finite yet complex decision space [3].

The main innovations of this paper are as follows: first, it achieves a unified modeling of statistical estimation and discrete optimization, directly incorporating uncertain parameters into the decision-making process; second, it constructs a structurally clear multi-stage decision-making framework that effectively describes quality propagation mechanisms; and third, it proposes a global optimization strategy based on full-space traversal, guaranteeing optimality within a computable scale. This method provides a general and scalable algorithmic paradigm for quality decision-making problems in multi-process production systems.

2. Sampling Estimation Method for Defective Rate

2.1. Model Establishment

During the procurement of parts and components, it is necessary to judge whether the actual defective rate exceeds the given nominal value. A hypothesis testing method is adopted for this judgment. Let the defective rate of parts and components be p, and the following test hypotheses are constructed: $H_0 : p \le 0.10$ and $H_1 : p > 0.10$. When the sample result is significantly large, the batch of parts can be considered to have quality risks.

2.2. Sample Size Calculation

To ensure the reliability of the test while controlling inspection costs, a reasonable minimum sample size needs to be determined. The following sample size estimation formula is adopted:

$$n = \left(\frac{Z_{\alpha/2}\sqrt{p(1-p)}}{E} \right)^2 \tag{1}$$

where $Z_{\alpha/2}$ is the critical value of the standard normal distribution, determined by the confidence level; p is the nominal defective rate, which is set to 0.10 here; E represents the allowable error, which is set to 0.05 here.

Under different confidence levels, the corresponding sample sizes are as follows:

When the confidence level is 95%: $n_{95} \approx 138$. When the confidence level is 90%: $n_{90} \approx 108$.

2.3. Result Explanation

According to the calculation results, the corresponding judgment rules can be obtained:

At the 95% confidence level, the batch of parts and components should be rejected when the sample defective rate is significantly higher than the nominal value;

At the 90% confidence level, the batch of parts can be accepted when the sample result does not obviously exceed the nominal value.

The defective rate estimation results obtained by the above method can be used as input parameters in the subsequent multi-process production decision-making model to describe the quality level at each stage.

3. Multi-Process Production Decision Model

3.1. Decision Structure

Overall, the production process can be divided into two stages: multiple parts are first assembled into semi-finished products, and then semi-finished products are assembled into final finished products. Whether inspection is performed during the process and whether non-conforming products are disassembled will directly affect the quality distribution and cost structure of subsequent stages [4].

Under this framework, there are 16 binary decision variables in total, each taking a value of 0 or 1 (representing "no inspection / no disassembly" and "inspection / disassembly" respectively). The corresponding decision space size is: $2^{16} = 65536$.

The meanings of each variable are as follows:

Whether to inspect parts 1–8: $x_{1\sim8}$; Whether to inspect semi-finished products 1–3: $x_{9\sim11}$; Whether to disassemble semi-finished products 1–3: $x_{12\sim14}$; Whether to inspect finished products: x_{15} ; Whether to disassemble non-conforming products: x_{16} .

Inspection decisions at different levels change the transmission path of defects, while disassembly decisions further affect resource recovery and the overall cost structure.

3.2. Decision Objective

The system objective is to minimize the total cost as much as possible under the premise of satisfying production constraints, which is equivalently to maximize the total profit. The objective function can be expressed as:

$$C_{\text{sum}} = V_{\text{sell}} + V_{\text{recycle-ban}} + V_{\text{recycle-cheng}} - C_{\text{ling}} - C_{\text{ban}} - C_{\text{cheng}} - C_{ci} \tag{2}$$

where each term corresponds to the cost of parts, semi-finished products, finished products and defect handling cost respectively, while taking sales revenue and recycling revenue from disassembly into account.

3.3. Model Construction

Since the decision space size is 2^{16} , which is still within the computable range, an exhaustive method is adopted to calculate the corresponding total cost for all decision combinations one by one, and then conduct comparison and screening.

3.3.1. Component Cost.

As the starting point of the production process, the cost of components consists of procurement cost and inspection cost. The introduction of inspection essentially represents a trade-off between early screening of defective items and increased inspection cost.

$$C_{\text{ling}} = C_{\text{gou}} + C_{\text{jian}},\ \ C_{\text{gou}} = N \times P_{\text{ling}},\ \ C_{\text{jian}} = N \times J_{\text{ling}} \times x \tag{3}$$

where x indicates whether inspection is performed. When $x = 1$, inspection cost is introduced, and the spread of defective items in subsequent stages is reduced. When $x = 0$, inspection expenses are saved, but quality risks continue to be passed downstream.

3.3.2. Semi-finished Product Cost.

Semi-finished products are in the middle layer of the assembly process. On the one hand, they inherit the quality of components, and on the other hand, they affect the quality of finished products, so inspection at this stage has a certain interception effect.

Its cost structure is similar to that of components, except that procurement is replaced by assembly:

$$C_{\text{ban}} = C_{\text{zu}} + C_{\text{jian}},\ \ C_{\text{zu}} = N \times P_{\text{zu}},\ \ C_{\text{jian}} = N \times J_{\text{jian}} \times x \tag{4}$$

where x corresponds to x_9 to x_{11}. Inspection at this stage can reduce defects entering the final assembly process, but will increase inspection costs at the same time.

3.3.3. Finished Product Cost.

The finished product stage mainly involves final assembly and finished product inspection, whose decisions directly affect the quality of products delivered to the market.

$$C_{\text{cheng}} = C_{\text{zhuang}} + C_{\text{jian}},\quad C_{\text{zhuang}} = N \times Z,\quad C_{\text{jian}} = N \times J_{\text{cheng}} \times x_{15} \tag{5}$$

Finished product inspection can prevent defective items from entering the market, but it also incurs additional costs, so it needs to be considered comprehensively with subsequent compensation and replacement costs.

3.3.4. Defect Handling Cost.

Defect handling is a key part of the cost structure, especially when inspection is insufficient, this part often rises significantly.

$$C_{ci} = C_{\text{pei}} + C_{\text{huan}} + C_{\text{chai}},\quad C_{\text{pei}} = N \times S,\quad C_{\text{huan}} = N \times H,\quad C_{\text{chai}} = N \times J_{\text{cheng}} \times x_{16} \tag{6}$$

Among them, compensation and replacement mainly come from non-conforming products at the market end, while disassembly decisions affect subsequent resource recovery and additional processing costs.

3.3.5. Revenue Component.

Revenue mainly comes from finished product sales and disassembly recycling.

Sales revenue is: $V_{\text{sell}} = N \times S$.

Recycling revenue from disassembly is:

$$V_{\text{recycle-ban}} = \sum N_i \times P_{\text{ling}i} \tag{7}$$

$$V_{\text{recycle-cheng}} = \sum N_i \times P_{\text{ban}i} \times R_i \tag{8}$$

It should be noted that not all semi-finished products obtained from finished product disassembly are necessarily usable, so a correction coefficient R_i is introduced to describe the proportion of usable parts, so as to avoid overestimation of recycling revenue.

3.4. Key Numerical Relationships

To unify the calculation scale, the initial procurement quantity of various components is set to 1, which transforms the problem into a unit cost analysis and facilitates comparison between different schemes.

At the same time, the effective quantity at each stage is affected by inspection decisions. The quantity entering the next stage after inspection is reduced according to the qualified rate, while all items enter subsequent processes without inspection. Therefore, in the assembly process, the minimum effective quantity among the components involved in assembly is usually taken as the actual input quantity to ensure consistency in the assembly process.

3.5. Optimal Decision Results

After traversing all 65536 combinations, several groups of decision schemes with optimal cost can be obtained, as shown in Table 1.

Table 1. Optimal decision combinations

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
1	1	1	0	1	0	1	1	1	1	1	1	1	1	0	1
1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	1
1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1

It can be seen that different decision combinations have certain structural differences, but all can achieve the same optimal cost level. This indicates that the optimal solution to the problem is not unique, but there exists a set of feasible solutions.

Further comparison shows that these schemes are highly consistent in key inspection links, while having certain flexibility in local decisions such as partial disassembly decisions.

Based on the above results, the properties of the model can be understood as a whole. The problem can be regarded as a combinatorial optimization problem within a finite decision space, where all inspection and disassembly decisions together form a discrete decision vector.

Since the decision space size is 2^{16}, the global optimal solution can be obtained by enumerating all possible schemes and calculating their corresponding total costs one by one. Different from continuous optimization problems, such problems focus more on the overall comparison between different decision combinations rather than local adjustment of a single variable.

4. Model Extension Considering Sampling Estimation Errors

In actual production, the defective rate cannot usually be obtained directly but is estimated through sampling inspection. Therefore, the assumption in the original model that treats the defective rate as a known constant has certain limitations.

On this basis, the model is extended: while keeping the original decision structure unchanged, the deterministic defective rate is replaced by parameters obtained from sampling estimation, and the optimal decision scheme is recalculated. This treatment is more consistent with the actual production environment and also enables the model to characterize a certain degree of uncertainty.

4.1. Resolving Under the Scenario of Two Types of Components

A relatively simplified scenario is first considered, namely a system containing only two types of components and finished products. Let the defective rates of component 1, component 2, and the finished product be b_1, b_2, and b_3 respectively. Under this setting, the system contains 4 decision variables, corresponding to component inspection, finished product inspection, and disassembly of non-conforming products respectively.

Definition of decision variables:

Whether to inspect component 1: x_1; Whether to inspect component 2: x_2; Whether to inspect finished product: x_3; Whether to disassemble non-conforming products: x_4.

The system objective is still to minimize the total cost, that is, to maximize the total revenue. The objective function is expressed as:

$$C_{\text{sum}} = V_{\text{sell}} + V_{\text{recycle}} - C_{\text{ling1}} - C_{\text{ling2}} - C_{\text{cheng}} - C_{\text{ci}} \tag{9}$$

where C_{ling1} and C_{ling2} represent the relevant costs of the two types of components, C_{cheng} represents the cost at the finished product stage, and C_{ci} represents the defect handling cost; V_{sell} and V_{recycle} represent sales revenue and disassembly recycling revenue respectively.

4.1.1. Model Construction.

The exhaustive method is still adopted in this part to calculate all decision combinations. The difference from the deterministic model is that the defective rate is given by estimated values, so the effective quantity and defective quantity at each stage vary with b_1, b_2, and b_3.

1. Component Cost

The cost of component 1 consists of procurement cost and inspection cost:

$$C_{\text{ling1}} = C_{\text{gou1}} + C_{\text{jian1}},\ \ C_{\text{gou1}} = N \times P_{\text{ling1}},\ \ C_{\text{jian1}} = N \times J_{\text{ling1}} \times x_1 \tag{10}$$

The inspection decision directly affects the effective quantity entering the subsequent assembly stage. If no inspection is performed, all components enter the next stage; if inspection is performed, only qualified parts are retained, so we have:

$$N_{\text{ling1}} = \begin{cases} N, & x_1 = 0 \\ (1-b_1)N, & x_1 = 1 \end{cases} \tag{11}$$

Component 2 is handled similarly:

$$C_{\text{ling2}} = C_{\text{gou2}} + C_{\text{jian2}},\ \ C_{\text{gou2}} = N \times P_{\text{ling2}},\ \ C_{\text{jian2}} = N \times J_{\text{ling2}} \times x_2 \tag{12}$$

$$N_{\text{ling2}} = \begin{cases} N, & x_2 = 0 \\ (1-b_2)N, & x_2 = 1 \end{cases} \tag{13}$$

Inspection increases the cost at the current stage but can reduce losses caused by defect propagation in subsequent stages, so it essentially represents a trade-off between local cost and overall cost.

2. Defect Handling Cost

Defect handling cost consists of three parts: compensation, replacement and disassembly.

$$C_{ci} = C_{pei} + C_{huan} + C_{chai},\ \ C_{pei} = N_{ci} \times S,\ \ C_{huan} = N_{ci} \times H,\ \ C_{chai} = N_{ci} \times J_{cheng} \times x_4 \tag{14}$$

where the calculation of the number of defective items N_{ci} is critical. Under uncertainty, defects may come from the finished product assembly process or from unfiltered non-conforming components, so its value is related to b_1, b_2, b_3 and inspection decisions x_1, x_2.

$$N_{ci} = \begin{cases} N_{cheng} \times b_3, & x_1 = 1, x_2 = 1 \\ N_{cheng} \times (1-b_1)b_3 + N_{cheng} \times b_1, & x_1 = 0, x_2 = 1 \\ N_{cheng} \times (1-b_2)b_3 + N_{cheng} \times b_2, & x_1 = 1, x_2 = 0 \\ N_{cheng} \times (1-b_1)(1-b_2)b_3 + N_{cheng} \times \left[1-(1-b_1)(1-b_2)\right], & x_1 = 0, x_2 = 0 \end{cases} \tag{15}$$

This expression characterizes the combined effects of component defects and assembly defects, making comparisons between different strategies more complete.

3. Revenue Component

Sales revenue is expressed as $V_{sell} = N \times S$. Recycling revenue from disassembly is:

$$V_{recycle} = N_1 \times P_{ling1} + N_2 \times P_{ling2} \tag{16}$$

Although disassembly incurs additional costs, it can offset part of the losses caused by defective items under certain conditions, so it should be considered together with the overall revenue.

4.1.2. Result Analysis and Optimal Scheme.

Assume the defective rates of component 1, component 2 and finished product are 0.1, 0.2 and 0.3 respectively. Corresponding to 4 decision variables, there are 16 combinations in total. The total revenue and cost of each scheme are calculated by exhaustive enumeration, and the results are shown in Table 2.

Table 2. Table of total revenue and total costs for each scenario

x_1	x_2	x_3	x_4	Total Profit	Component 1 Cost	Component 2 Cost	Finished Product Cost	Disassembly/ Replacement Cost	Sales Revenue	Component 1 Residual Value	Component 2 Residual Value
0	1	1	0	-3.976	4	21	7.2	0	28.224	0.864	5.328
1	0	1	0	-3.876	6	18	8.1	0	28.224	1.584	3.888
1	0	0	0	-3.552	6	18	5.4	24.552	50.4	1.584	3.888
0	1	0	0	-3.352	4	21	4.8	18.352	44.8	0.864	5.328
1	1	1	0	-2.84	6	21	7.2	0	31.36	0.96	4.32
0	0	1	0	-2.776	4	18	9	0	28.224	1.584	5.328
0	0	0	0	-2.752	4	18	6	30.752	56	1.584	5.328
1	1	0	0	-1.88	6	21	4.8	14.88	44.8	0.96	4.32
1	0	1	1	-0.384	6	18	8.1	1.98	28.224	1.584	3.888
1	0	0	1	-0.06	6	18	5.4	26.532	50.4	1.584	3.888
0	1	1	1	0.736	4	21	7.2	1.48	28.224	0.864	5.328
1	1	1	1	1.24	6	21	7.2	1.2	31.36	0.96	4.32
0	1	0	1	1.36	4	21	4.8	19.832	44.8	0.864	5.328
0	0	1	1	1.656	4	18	9	2.48	28.224	1.584	5.328
0	0	0	1	1.68	4	18	6	33.232	56	1.584	5.328
1	1	0	1	2.2	6	21	4.8	16.08	44.8	0.96	4.32

It can be seen from the table that the differences between different strategies are obvious. Reducing inspections lowers upfront costs but leads to higher subsequent replacement and disassembly costs. Properly increasing inspection and disassembly improves overall revenue despite higher local costs.

Under the current parameter settings, the optimal decision combination is $(x_1, x_2, x_3, x_4) = (1,1,0,1)$, with a total revenue of 2.2, which is the best result among all combinations. This shows that inspecting key components combined with disassembly of non-conforming products, without inspecting finished products, achieves a better cost–benefit balance under these parameters.

This result also indicates that, under parameter uncertainty, rationally allocating inspection and disassembly strategies at different stages is more effective than simply increasing inspection intensity.

4.2. Resolving for the Multi-Process Scenario

In this section, the multi-process production decision model retains its original structure, but the defective rates at each stage are no longer regarded as known constants. Instead, they are introduced into the model as estimated values obtained from sampling inspection. The model itself remains unchanged, and the variation mainly lies in the input parameters. This treatment is more consistent with actual production decision conditions and provides a more realistic explanatory basis for the model [5].

Let the defective rates of 8 components be b_1 to b_8, the defective rates of 3 semi-finished products be b_9 to b_{11}, and the defective rate of the finished product be b_{12}. Under this setting, the system still requires joint decision-making on inspection and disassembly strategies.

4.2.1. Decision Scheme.

The whole production process is still divided into two levels: multiple components are first assembled into semi-finished products, and then semi-finished products are assembled into final finished products. Since defects may exist in components, semi-finished products and finished products, inspection decisions at different stages directly affect the transmission mode of defects in subsequent processes. Whether non-conforming products are disassembled further affects processing costs and recycling revenue [6].

Therefore, this section still includes 16 binary decision variables, and the corresponding decision space size is $2^{16} = 65536$.

It can be seen from this structure that front-end inspection determines whether defects continue to flow into subsequent assembly stages, intermediate inspection determines the quality interception capability at the semi-finished product level, while end inspection and disassembly mostly affect market risks and resource recovery.

4.2.2. Decision Objective.

The objective of this section is consistent with the deterministic scenario, which is still to find the scheme with the lowest total cost, that is, the highest total profit, by comparing the total revenue under different decision combinations. The objective function is written as:

$$C_{\text{sum}} = V_{\text{sell}} + V_{\text{recycle-ban}} + V_{\text{recycle-cheng}} - C_{\text{ling}} - C_{\text{ban}} - C_{\text{cheng}} - C_{ci} \tag{17}$$

where C_{ling} represents component-related costs, C_{ban} represents semi-finished product stage costs, C_{cheng} represents finished product stage costs, and C_{ci} represents defect handling costs; V_{sell} is sales revenue, and $V_{\text{recycle-ban}}$ and $V_{\text{recycle-cheng}}$ represent recycling revenue from semi-finished product disassembly and finished product disassembly respectively.

4.2.3. Decision Model.

This part is still essentially a multi-stage combinatorial optimization problem. The 16 decision variables correspond to 65536 combinations. Although the scale is large, it is still within the range of direct traversal. Therefore, the exhaustive method is still adopted to calculate the total revenue of each decision scheme one by one, and then select the optimal results accordingly.

(1) Component Cost

For each type of component, its cost still consists of two parts: procurement cost and inspection cost:

$$C_{\text{ling}} = C_{\text{gou}} + C_{\text{jian}},\quad C_{\text{gou}} = N \times P_{\text{ling}},\quad C_{\text{jian}} = N \times J_{\text{ling}} \times x \tag{18}$$

where x indicates whether the component is inspected. Inspection increases the cost at the current stage, but also reduces the probability of defective items flowing into subsequent processes. Thus, it is not a pure additional expenditure, but a pre-control of subsequent losses.

Different from the deterministic scenario, the effective quantity of components here is no longer determined by a fixed defective rate, but given by the estimated value b_i. Therefore, the quantity of the i-th type of component entering the subsequent stage can be expressed as:

$$N_{\text{ling}i} = \begin{cases} N, & x_i = 0 \\ (1-b_i)N, & x_i = 1 \end{cases} \tag{19}$$

In other words, all items are assumed to enter the next stage without inspection; only qualified parts are retained after inspection.

(2) Semi-finished Product Cost

Semi-finished products are in the middle assembly layer, which inherits the quality of components and further affects the quality of finished products, so inspection decisions at this stage are usually critical. Its cost consists of assembly cost and inspection cost:

$$C_{\text{ban}} = C_{\text{zu}} + C_{\text{jian}},\ \ C_{\text{zu}} = N \times P_{\text{zu}},\ \ C_{\text{jian}} = N \times J_{\text{jian}} \times x \tag{20}$$

where x corresponds to x_9 to x_{11}. Inspection at the semi-finished product stage can cut off part of the defect propagation before final assembly, but will also bring additional inspection costs accordingly.

(3) Finished Product Cost

The cost structure at the finished product stage is relatively simple, mainly consisting of final assembly cost and finished product inspection cost:

$$C_{\text{cheng}} = C_{\text{zhuang}} + C_{\text{jian}},\ \ C_{\text{zhuang}} = N \times Z,\ \ C_{\text{jian}} = N \times J_{\text{cheng}} \times x_{15} \tag{21}$$

This stage directly corresponds to market output, so whether to inspect finished products is essentially a balance between current inspection costs and market-side risks.

(4) Defect Handling Cost

Defect handling cost includes three parts: compensation, replacement and disassembly:

$$C_{ci} = C_{\text{pei}} + C_{\text{huan}} + C_{\text{chai}},\ \ C_{\text{pei}} = N \times S,\ \ C_{\text{huan}} = N \times H,\ \ C_{\text{chai}} = N \times J_{\text{cheng}} \times x_{16} \tag{22}$$

Defect handling cost is a relatively sensitive part of the whole model. When inspection is insufficient, this item usually rises rapidly. Therefore, the optimal scheme does not necessarily correspond to the least inspection, but often depends on whether the inspection configuration is placed at a more appropriate position.

(5) Semi-finished Product Disassembly Cost

If semi-finished products are judged as unqualified after inspection, they may enter the disassembly process. The corresponding cost can be written as:

$$C_{\text{chai}} = N \times J_{\text{cheng}} \times x \tag{23}$$

where x corresponds to x_{12}, x_{13} or x_{14}. Although this part increases processing costs, it also provides conditions for subsequent component recycling.

(6) Sales Revenue

Sales revenue remains unchanged and can be expressed as $V_{\text{sell}} = N \times S$.

(7) Disassembly and Recycling Revenue

Recycling revenue is still divided into two parts.

The first part comes from recycling components obtained by disassembling semi-finished products:

$$V_{\text{recycle-ban}} = \sum N_i \times P_{\text{ling}i} \tag{24}$$

It is assumed that all components obtained after disassembly can be reused, so their recycling value can be calculated directly based on quantity and unit price.

The second part comes from recycling semi-finished products obtained by disassembling finished products:

$$V_{\text{recycle-cheng}} = \sum N_i \times P_{\text{bani}} \tag{25}$$

However, this part cannot be converted directly by the number of semi-finished products, because the semi-finished products obtained from disassembly may still have defects. Therefore, a correction coefficient R_i is introduced to represent the proportion of usable parts. The revised revenue expression is:

$$V_{\text{recycle-cheng}} = \sum N_i \times P_{\text{bani}} \times R_i \tag{26}$$

This correction is necessary; otherwise, recycling revenue would be overestimated and the results would be overly optimistic.

4.2.4. Optimal Decision Scheme.

In this section, the defective rates of the 8 components are set as [0.05, 0.10, 0.05, 0.20, 0.05, 0.25, 0.15, 0.15], the defective rates of the 3 semi-finished products are [0.15, 0.15, 0.15], and the finished product defective rate b_{12} is set to 0.20.

After exhaustively calculating the total revenue for all combinations of the 16 decision variables, the optimal scheme with the lowest total cost is obtained in the Table 3:

Table 3. Optimal decision scheme

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_1	x_2	x_3	x_4
0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1

It can be seen that under this set of parameters, the optimal scheme does not apply a uniform strategy to all links, but adopts selective inspection and disassembly for some components and subsequent stages. This result is reasonable because, in a multi-process scenario, the value of inspection varies at different positions. Some links are more suitable for pre-control, while others are better compensated by subsequent disassembly.

From the results, this set of strategies achieves a good balance among inspection cost, defect handling cost and recycling revenue, thus corresponding to the lowest total cost. It also shows that when the defective rate is estimated by sampling with a certain degree of uncertainty, multi-stage joint decision-making is more effective than adjusting a single link alone.

5. Conclusion

This paper investigates quality control and cost optimization in multi-process production systems by developing a unified framework that combines sampling estimation with discrete decision optimization. The production process is represented as a decision space with 16 binary variables and 65,536 possible combinations, enabling systematic modeling and global optimization of inspection and disassembly strategies. Results show that, under multi-stage quality propagation and cost coupling, properly allocating inspection and resource recovery strategies at key stages can improve overall profitability more effectively than strengthening control at only one stage. Methodologically, the study integrates uncertainty parameters directly into the decision model, overcoming the traditional separation between statistical analysis and optimization. The framework also shows good generality and can serve as a reference for multi-level decision-making in complex systems. Future research could further incorporate intelligent optimization algorithms and dynamic data updating to improve applicability in larger-scale and real-time scenarios.

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