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Emergency Responder Route Optimization System: A Mathematical Framework for Building Safety Sweeps

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Abstract. In emergency scenarios such as fires, earthquakes, or active shooter incidents, the rapid and safe searching of multi-story buildings is critical for saving lives. This paper presents a mathematical framework for optimizing emergency responder routes during building safety sweeps. The proposed system transforms floor plans into weighted graph representations, where nodes denote rooms, hallways, stairwells, and exits, and edges represent traversable paths with associated travel and service times. A dynamic risk assessment module is developed to prioritize rooms based on type, occupancy, floor level, and real-time hazard propagation, such as fire or smoke spread. The routing problem is formulated as a variant of the vehicle routing problem with time windows and solved using constraint programming integrated with k-means clustering for exit assignment. The model minimizes total mission completion time while ensuring coverage of all rooms and adherence to safety constraints. Case studies involving single-floor offices, two-story mixed-use buildings, and high-risk facilities demonstrate the system's effectiveness. Results show that optimal team sizes range from two to five responders depending on building complexity and emergency severity, with mission completion times varying between 1,477 seconds under normal conditions and over 3,500 seconds under dynamic fire spread. The framework also incorporates realistic extensions such as cascading failures, congestion dynamics, and technology integration. This work provides a practical, data-driven tool for emergency planners to enhance responder efficiency and occupant safety.

Keywords: Emergency response optimization; Building evacuation; Graph theory; Risk assessment; Constraint programming; Vehicle routing problem.

1. Introduction

First responders must rapidly search buildings during emergencies to ensure complete evacuation. Speed directly impacts survival rates, yet complex layouts, dynamic hazards, and safety constraints make efficient sweeping challenging. The routing optimization problem addressed in this paper shares conceptual foundations with broader emergency medical service routing literature [5].

This paper develops a mathematical framework that transforms floor plans into navigable graphs and generates optimal multi-responder routes. The model accounts for travel times, room-specific service durations, dynamic risk propagation (e.g., fire spread), and operational constraints including time windows and skill requirements. Our objective is to minimize total mission completion time while ensuring full coverage and responder safety.

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2. Assumptions

To simplify the problem, we assume that the floor layout accurately depicts the actual building and the layout of building does not change during fire, because otherwise the actual route will be unpredictable. Also, we assume all firefighters' speed and proficiency follow our data, and they receive and strictly executed the route plan.

3. Model Framework Overview

We develop a graph-theoretic optimization framework for emergency response in multi-story buildings. The framework consists of three integrated components:

1. Graph representation: Building spaces are abstracted as a weighted network capturing rooms, corridors, stairwells, exits, and their traversable connections.

2. Dynamic risk assessment: Room priorities and accessibility evolve with emergency progression (e.g., fire spread), ensuring routes adapt to changing hazard conditions.

3. Route optimization: A constraint programming model generates coordinated search plans that minimize total completion time while respecting safety constraints, room priorities, and responder capabilities.

The framework outputs optimal team size, mission completion time, and step-by-step route sequences for each responder.

4. Graph Layout Model

We abstract the building as a weighted undirected graph $\mathrm{G} = (\mathrm{V}, \mathrm{E})$, where nodes represent rooms, stairwells, hallways, and exits, and edges represent traversable paths, and edge weights represents travel distance. For each node $\mathrm{i} \in \mathrm{V}$,we define a set of attributes to capture its geometric, functional, and risk-related properties:

- Location: (x_i, y_i, z_i) coordinates and floor level,
- Area: $Ai \in \mathbb{R}^{+}$(in m^2), physical size of the room, proportional to sweep time,
- Type τ_i: Office, laboratory, childcare, stairwell, exit
- Priority p_i: Importance level (1–10) for sweeping,
- Time-dependent risk $\rho_i(\mathrm{t}) \in [0,1]$: Hazard progression (fire, smoke).

For same-floor travel in open layouts, distance is computed using Manhattan distance: $d_{ij} = \mid x_i - x_j \mid + \mid y_i - y_j \mid$.

For complex layouts with corridors, responders cannot move freely between rooms, and Manhattan distance underestimates actual travel paths. We refine the graph by introducing hallway intersection nodes $h \in V$ that represent corridor junctions and turns. Each room, stairwell, and exit node connects only to its nearest hallway intersections. The travel distance d_{ij} between any two nodes i and j is computed as the shortest path through the corridor network: $\mathrm{d_{ij}} = \min_{\mathrm{p} \in \mathcal{P}_{\mathrm{ij}}} \sum_{(\mathrm{u},\mathrm{v}) \in \mathrm{p}} \mathrm{d}\,(\mathrm{u}, \mathrm{v})$, where $\mathcal{P}_{\mathrm{ij}}$ is the set of all feasible paths from i to j traversing hallway intersections, and $\mathrm{d}(\mathrm{u}, \mathrm{v})$ is the distance along edge (u, v). We precompute all-pairs shortest paths using the Floyd-Warshall algorithm, enabling constant-time distance lookups during optimization.

For inter-floor travel, responders must use stairwells. The distance between nodes on different floors is:

$$d_{ij} = \min_{s \in V_{stairwell,i}} [d_{i,s} + h_{z_i, z_j} + d_{s,j}]$$

where $V_{stairwell,i}$ are set of stairwell nodes on floor z_i, $h_s(z_i, z_j) = |z_i - z_j| \cdot H_{floor}$ is vertical travel distance.

4.1. Dynamic Risk and Accessibility

The static building graph $G = (V, E)$ serves as the foundation, but emergency conditions introduce dynamic modifications to both node accessibility and edge traversability.

4.1.1. Time-Dependent Risk Propagation

Each node $i \in V$ possesses a time-dependent risk level $\rho_i(t) \in [0,1]$ that evolves according to emergency progression. For fire scenarios, we use a modified sigmoidal growth model:

$$\rho_i(t) = (1 - e^{-t/180}) \cdot \frac{1}{1 + e^{-0.006(t - t_{i,critical})}}$$

The first term captures rapid smoke and heat propagation (dangerous conditions develop within 3-5 minutes). The second term controls flashover progression, with risk rising from 25% to 75% in approximately 230 seconds. The multiplicative coupling ensures rapid initial hazard exposure accelerates subsequent flashover development. This parametric model is consistent with established fire dynamics principles and can be further refined through coupling with computational fluid dynamics simulations for more accurate hazard prediction [1] [9] [10].

The critical time $t_{i,critical}$ is the time when node i reaches 50% risk saturation. The critical time depends on room-specific factors:

$$t_{i,critical} = t_{i,fire-start} + \frac{A_i}{A_0} \cdot r(\tau_i) \cdot m_{spread} \cdot \phi_{z_i}$$

Here, $A_0 = 50m^2$ is selected based on parametric fire curve studies, which identify this value as representative of typical office room dimensions for engineering fire safety analysis [2] [3]. A_i is the floor area of room i (m2), $r(\tau_i)$ is the room-type flammability factor (varying by room function, e.g., high-fuel laboratories have higher values than hallways), m_{spread} is the emergency-scaled spread multiplier, and ϕ_{z_i} is the floor-specific risk multiplier (increasing with floor height due to evacuation challenges).

The model ensures $\rho_i(0) = 0$ while capturing realistic fire progression timelines, with high-risk rooms reaching dangerous levels ($\rho_i > 0.75$) within 5-8 minutes and safer areas remaining viable ($\rho_i \leq 0.75$) for 15-20 minutes.

4.1.2. Edge Weight Modification by Hazard Propagation

Travel difficulty increases with hazard exposure. The time-dependent edge weight (hazard-dcaled distances) is defined as:

$$w_{ij}(t) = d_{ij} \cdot \left[1 + \alpha \cdot \max(\rho_i(t), \rho_j(t))\right] + \beta \cdot \int_{P_{ij}} \rho(x,t)dx$$

with $\alpha = 0.5$ (endpoint hazard sensitivity) and $\beta = 0.3$ (path-integrated exposure cost).

The first term accounts for reduced traversal speed when entering or exiting hazardous areas. The second term models cumulative risk accumulated while traversing the shortest path from node i to node j.

For computational implementation, the path integral is approximated using the trapezoidal rule along the shortest path.

$$\int_{P_{ij}} \rho(x,t)dx \approx \sum_{k=1}^{n-1} \frac{\rho_{v_k}(t) + \rho_{v_{k+1}}(t)}{2} \cdot d_{v_k, v_{k+1}}$$

$v_1, v_2, \ldots, v_n$ are the consecutive nodes constituting the shortes path p_{ij}, with $v_1 = i$ and $v_n = j$. The dynamic edge weights $w_{ij}(t)$ provide a safety-aware cost metric for generating routes that avoid hazardous areas.

4.1.3. Node Accessibility Dynamics

Node accessibility $\psi_i(t) \in [0,1]$ models the progressive exclusion of hazardous areas from operational planning as emergencies evolve. This critical safety mechanism transitions rooms from fully accessible to completely blocked based on real-time risk assessment

$$\psi_i(t) = \begin{cases} 1, & t < t_{i,block} \\ 0, & t \geq t_{i,block} \end{cases}$$

The function operates in two distinct phases. Initially, rooms remain fully accessible ($\psi_i(t) = 1$) until the blocking time $t_{i,block}$ exceeds operational safety limits: $t_{i,block} = \min\{t: \rho_i(t) > \rho_{safe}\}$.

Here, $\rho_{safe} = 0.75$ is the maximum acceptable risk level for responder entry. This ensures that dangerous areas are quickly removed from consideration, preventing the optimization from routing responders through life-threatening conditions.

4.1.4. Dynamic Graph Representation

The static building graph $G = (V, E)$ transforms into a time-dependent graph $G(t) = (V(t), E(t))$ that dynamically adapts to emergency progression:

$$V(t) = \{i \in V: \psi_i(t) = 1\}$$
$$E(t) = \{(i,j) \in E: \psi_i(t) = 1 \; and \; \psi_j(t) = 1\}$$

where $\psi_i(t) \in \{0,1\}$ is the node accessibility status defined in Section 4.1.3.

This adaptive structure serves three crucial safety functions:

1. **Node exclusion:** Only accessible nodes ($\psi_i(t) = 1$) remain in the operational graph. Once a room's risk exceeds the safety threshold ($\rho_i(t) > \rho_{safe} = 0.75$), it will be removed from consideration.

2. **Edge exclusion:** Edges require both endpoints to be accessible, preventing routes that would enter or pass through hazardous zones.

3. **Dynamic edge costs:** For remaining edges, travel costs are modified by time-dependent weights w_{ij}(t) as defined in Section 4.1.2, reflecting increased sweeping cost and increased risk exposure in hazardous areas.

This dynamic graph formulation ensures that generated routes remain operationally feasible as conditions deteriorate, automatically excluding dangerously compromised areas while accounting for increased traversal difficulty in active hazard zones.

4.2. Travel Time Estimation

Travel time between nodes depends on distance, movement speed, and emergency conditions. Studies on heat transfer in steel-concrete composite structures provide important context for understanding how fire conditions may affect building structural integrity and, consequently, responder movement paths [2].

For horizontal movement on the same floor, travel time is:

$$t_{ij}^{horiz} = \frac{w_{ij}(t)}{v_{walk}}$$

where $w_{ij}(t)$ is the time-dependent edge weight incorporating both physical distance and risk (Section 4.1) and $v_{walk} = 1.4\, m/s$ is normal walking speed. Door processing adds $Y_{door} = 10$ seconds per entry/exit.

For vertical movement between floors, responders must use stairwells. The vertical component uses physical distance rather than hazard-scaled distance, as stairwell risk is captured separately:

$$t_{ij}^{vert} = \frac{h_s(z_i, z_j)}{v_{stairs}} + Y_{stairs}$$

where $h_s(z_i, z_j) = |z_i - z_j| \cdot H_{floor}$ is vertical distance, $H_{floor} = 4$ m, $vstairs = 0.1367$ m/s [12].

Total Path Travel Time. For a complete path $p = (i, v1, v2, \ldots, vm, j)$, total travel time is:

$$t_{path}(p, E, t) = \sum_{(u,v)\in p} t_{uv}(E, t)$$

4.3. Service Time Estimation

The time required to properly search and verify a room depends on its physical characteristics, function, and emergency conditions.

Base Service Time. For a room i, the base service time is proportional to its area and room-specific sweep rate:

$$s_i^{base} = A_i \cdot \alpha(\tau_i)$$

where A_i is room area (m²) and $\alpha(\tau_i)$ is the sweep rate coefficient (sec/m²) based on room type. Different room functions require varying inspection intensities.

Table 1. Room Sweep Rate Coefficients

Room Type	α(τ) (s/m2)	**Justification**
Office	10	Standard visual verification
Conference	8	Open layout, quick assessment
Laboratory	20	Hazardous materials, detailed inspection
Childcare	12	Careful search for children, hiding spots
Storage	15	Cluttered environment, thorough check
Restroom	5	Small space, rapid verification
Warehouse	25	Large area, complex layout
Hallway	5	Corridor sweep, visual line-of-sight

Emergency Adjustment. Under emergency conditions, service times increase due to reduced visibility, protective equipment, and heightened caution:

$$s_i(\mathcal{E}, t) = s_i^{base} \cdot \beta(\mathcal{E}) \cdot \psi_i(t)$$

where $\beta(\mathcal{E})$ is the emergency-specific service multiplier and $\psi_i(t) \in \{0,1\}$ is node accessibility. If a room becomes inaccessible ($\psi_i(t)=0$), it is removed from the search plan.

Table 2. Emergency Impact Multipliers

Emergency Type $\mathcal{E}$	$\beta(\mathcal{E})$ (Service)	**Rationale**
Fire	1.3	Reduced visibility, protective gear
Gas Leak	1.1	Respiratory protection, cautious movement
Earthquake	1.5	Structural damage, debris navigation
Active Shooter	1.2	Tactical movement, threat assessment
Normal	1.0	Standard operating conditions

Special Area Service Times. For non-room nodes (hallways, stairwells, exits), service times are fixed constants rather than area-dependent:

$$s_j(\mathcal{E}) = \begin{cases} Y_{hall}, & \text{j is hallway} \\ Y_{stairs}, & \text{j is stairs} \\ Y_{exit}, & \text{j is exit} \end{cases}$$

with $Y_{hall} = 5\ s,\ Y_{stairs} = 15\ s,$ and $Y_{exit} = 0\ s.$

4.4. Combined Time Model

The total cost of moving from node i to j and searching node j is:

$$c_{ij}(\mathcal{E}, t) = t_{ij}(\mathcal{E}, t) + s_j(\mathcal{E}, t)$$

where $t_{ij}(\mathcal{E}, t)$ is travel time (Section 4.2) and $s_j(\mathcal{E}, t)$ is service time (Section 4.3). For inaccessible nodes ($\psi_j(t) = 0$), set $c_{ij} = \infty$.

For a responder path $p = (v_1, v_2, \ldots, v_m)$, total mission time is: $T_{path}(p, \mathcal{E}, t) = \sum_{k=1}^{m-1} c_{v_k v_{k+1}}(\mathcal{E}, t)$.

The overall makespan across all responders Kis:$T_{max} = \max_{k \in K} T_{path}(p_k, \mathcal{E}, t)$.

Minimizing T_{max} is the optimization objective (Section 5).

5. Responder Routing Optimization Model

5.1. Decision Variables

We define the following decision variables for our optimization model:

$$x_{ij}^k = \begin{cases} 1 & if\ responder\ k\ moves\ from\ node\ i\ to\ node\ j \\ 0 & otherwise \end{cases}$$
$$y_i^k = \begin{cases} 1 & if\ node\ i\ is\ served\ by\ responder\ k \\ 0 & otherwise \end{cases}$$
$$t_i^k = arrival\ time\ of\ responder\ k\ at\ node\ i$$
$$T_{max} = maximum\ completion\ time\ among\ all\ responders$$

5.2. Objective Function

Our primary objective is to minimize the maximum mission completion time across all responders:

$$\min T_{max}$$

This ensures the earliest possible completion of the building sweep.

5.3. Constraints

Room Coverage Constraints

Every room must be visited by exactly one firefighter: $\sum_{k \in K} y_i^k = 1 \forall i \in R$ where R is the set of all rooms requiring inspection. This constraint makes sure that no room is overlooked during the search and rescue operation while preventing redundant visits.

Flow Conservation Constraints

Each responder's route forms a continuous path:

$$\sum_{j \in V} x_{ij}^k = y_i^k \forall i \in V, k \in K$$

$$\sum_{i \in V} x_{ij}^k = y_j^k \forall j \in V, k \in K$$

These constraints also ensure that each firefighter enters and exits every visited node exactly once, forming continuous in the building. The second constraint connects the movement variables with the service assignment variables for consistency.

Exit Constraints.

Each firefighter must begin and conclude their route at designated exit points:

$$\sum_{e \in E_{start}^k} \sum_{j \in V} x_{ej}^k = 1 \forall k \in K$$

$$\sum_{e \in E_{end}^k} \sum_{i \in V} x_{ie}^k = 1 \forall k \in K$$

E^k_{start} and E^k_{end} represent the assigned starting and ending exits for firefighter k. The starting and ending exits are determined through the k-means clustering procedure described in Section 6.3.

Time Windows Constraints

High-priority rooms must be reached by their deadlines:

$$t_i^k + s_i \leq L_i \forall i \in R, k \in K$$

Where t_i^k is the arrival time of responder k at room i, s_i is the service time at room i, and L_i is the latest allowable arrival time for room i.

6. Solution Procedure

6.1. K-means Exit Assignment Method

To reduce problem complexity and balance workloads, we first assign responders to building exits based on room proximity. Using k-means clustering with Manhattan distance, rooms are partitioned into | K | clusters:

$$\mathcal{C} = C_1, C_2, \ldots, C_{|K|} = KMeans(R, |K|, metric = d_M)$$

where clusters C_k are determined by minimizing the sum of squares within-clusters [7].

Each reponders assigned a the cluster and enters/exits the building from the entrance/exit in their designated cluster, ensuring spatially coherent search zones that minimize cross-building travel and stairwell transitions.

6.2. Optimization Algorithm

We implement the routing model using Google OR-Tools with constraint programming, selected for its effectiveness in solving vehicle routing problems with complex constraints [6].

This optimization model generates operational plans including starting positions, optimal number of responders, route sequence and total time of completion.

Algorithm 1 Sweeping Routing Optimization

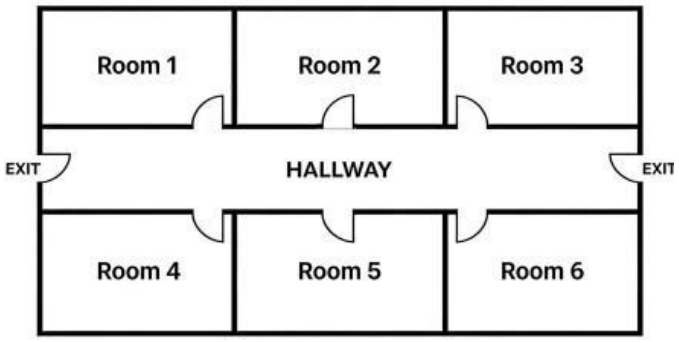
Input: Building layout, emergency types, target time Ttarget
Output: Optimal routes, team size, completion time
Step 1. Zone Assignment
    └── Assign rooms and exits to firefighters via k-means clustering
Step 2. Route Optimization
    ├── Initialize routing model with |K| responders
    ├── Set start/end exits for each firefighter
    ├── Define cost matrix c_ij = travel time + service time
    ├── Add constraints
    └── Solve using guided local search
Step 3. Team Size Check
    └── If makespan > T_target, increase |K| and repeat

7. Case Studies and Application

We evaluate our framework across three building scenarios of increasing complexity. Table 3 summarizes the key parameters and results.

7.1. Scenario 1: One-Floor Office

Setup. Six offices (48 m² each) on one floor with two exits (West and East). $V = \{R_1, R_2, R_3, R_4, R_5, R_6, E_W, E_E\}$. Normal walking speed 1.4 m/s, distance matrix is constructed based on Manhattan distances , estimated service time 483 s per room (normal) and 627 s (emergency). The optimization model was executed with target completion time T_{target} = 3600 seconds.



Results. Optimal team size is 2 firefighters. K-means clustering partitions rooms into {R1, R2, R3} and {R4, R5, R6}, assigned to West and East exits respectively. This spatial partitioning minimized the total travel distance while ensuring balanced workload distribution. The optimal route generated are:

Responder 1: Ew → R1 → R2 → R3 → Ew

Responder 2: EE → R6 → R4 → R5 → EE

Completion time: 1,477 s (normal), 1,894 s (emergency). Adding a third firefighter reduces time by only 8%, confirming 2 as optimal (see Figure 1 below).

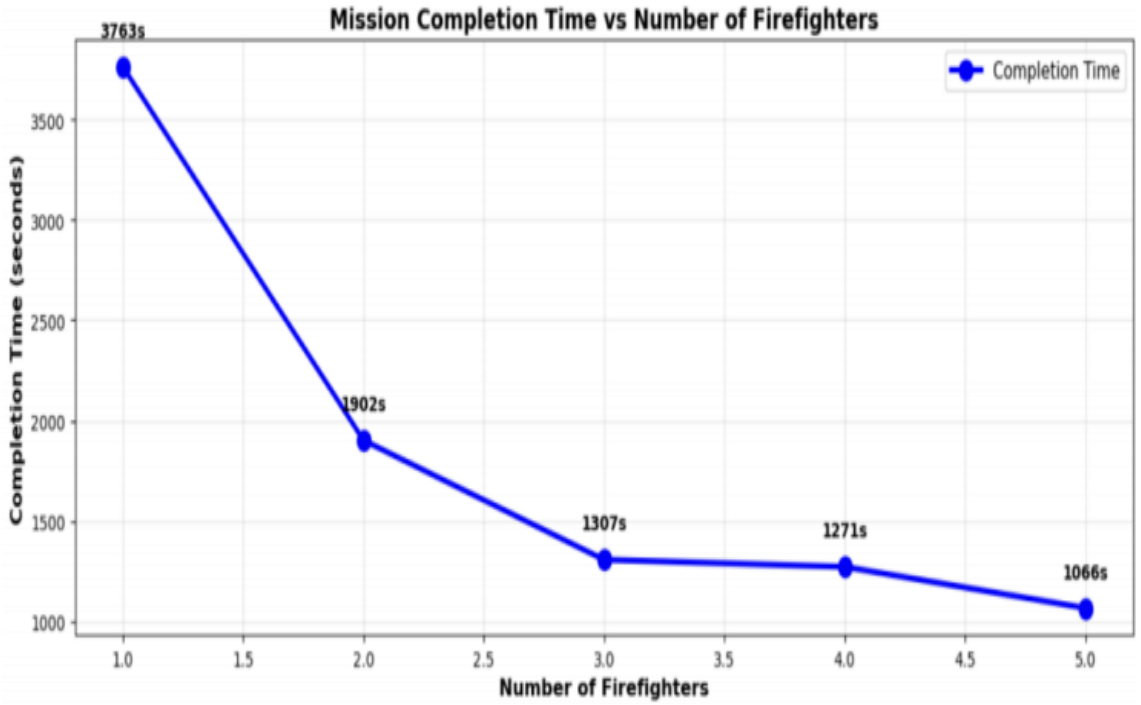


Figure 1. Comletion time vs number of responders

7.2. Four-Floor Mixed-Use Building

Setup. Four floors with diverse room types. The first floor has direct access to both exits Ew and EE, but the other floors must use west or east stairs to reach the first floor exits. Stairwells connect floors (vertical speed 0.1367 m/s, 15 s processing). Fire emergency multiplier β(fire) = 1.3.

Table 3. Room information for Floor i

Floor i			
Room	Type	Area(m2)	Service Time(s)
R1_ i	Office	48	627
R2_ i	Conference	60	627
R3_ i	Lab	80	1043
R4_ i	Storage	50	653
R5_ i	Childcare	100	1173
R6_ i	Restroom	20	133

Results. Optimal team size: 2 firefighters. Total completion time: 3,538 s (28% increase from normal due to emergency multipliers). Routes coordinate stairwell usage and prioritize high-risk rooms (childcare, lab) with longer service times.

Optimal Routes:
Responder 1 (West Exit):

$E_W \rightarrow$ Stair_W_1$\rightarrow$ Stair_W_3$\rightarrow$ R1_3$\rightarrow$ R2_3 $\rightarrow$ R3_3 $\rightarrow$ Stair_E_3 $\rightarrow$ Stair_E_2$\rightarrow$ R3_2 $\rightarrow$ R2_2 $\rightarrow$ R1_2$\rightarrow$ Stair_W_2 $\rightarrow$Stair_W_1 $\rightarrow E_W$

Responder 2 (East Exit):

$E_E \rightarrow$ Stair_E_1$\rightarrow$ Stair_E_2$\rightarrow$ R3_2$\rightarrow$ R2_2 $\rightarrow$ R1_2 $\rightarrow$ Stair_W_2 $\rightarrow$ Stair_W_4$\rightarrow$ R1_4 $\rightarrow$ R2_4 $\rightarrow$ R3_4$\rightarrow$ Stair_E_4 $\rightarrow$Stair_E_1 $\rightarrow E_E$

The total time taken exceeds about 28% of total time taken without emergency multipliers. However, this significantly improve the reliability of saftiness because firefighters stay longer time in room that required more ensurance in safety, like the Childcare one and the Lab.

7.3. High-Risk Facility with Time Windows and Fire Spread

Setup. Two-floor layout with priority-based classification. Rooms ranked by priority score (area × type coefficient × floor multiplier). Three priority levels with deadlines:

- High priority: 900 s (childcare, lab)
- Medium priority: 1,800 s (office, conference, storage)
- Low priority: 2,700 s (restroom)

Recell hazard propagation model: risk evolves as $\rho_i(t) = \frac{1-e^{-t/180}}{1+e^{-0.006(t-t_{i,critical})}}$

with room-specific critical times (300 s for high priority rooms, 600 s for medium priority, 1,200 s low priority). Rooms become inaccessible when $\rho_i(t) > 0.75$.

Results. Optimal team size: 5 firefighters (increased from 2 due to time windows and fire constraints). Completion time: 3111 s, meeting all deadlines before critical rooms become inaccessible at 3451 s.

Optimal Routes:
Responder 1 (West Exit): E_W → R3_1 (lab) → R1_1 → R2_1 → R4_1 → E_W

Responder 2 (East Exit): E_E → R5_1 (childcare) → R6_1 → E_E

Responder 3 (West Exit) : E_W → stair → R3_2 (lab) → R1_2 → R2_2 → R4_2 → stair → E_W

Responder 4 (East Exit): E_E → stair → R5_2 (childcare) → R6_2 → stair → E_E

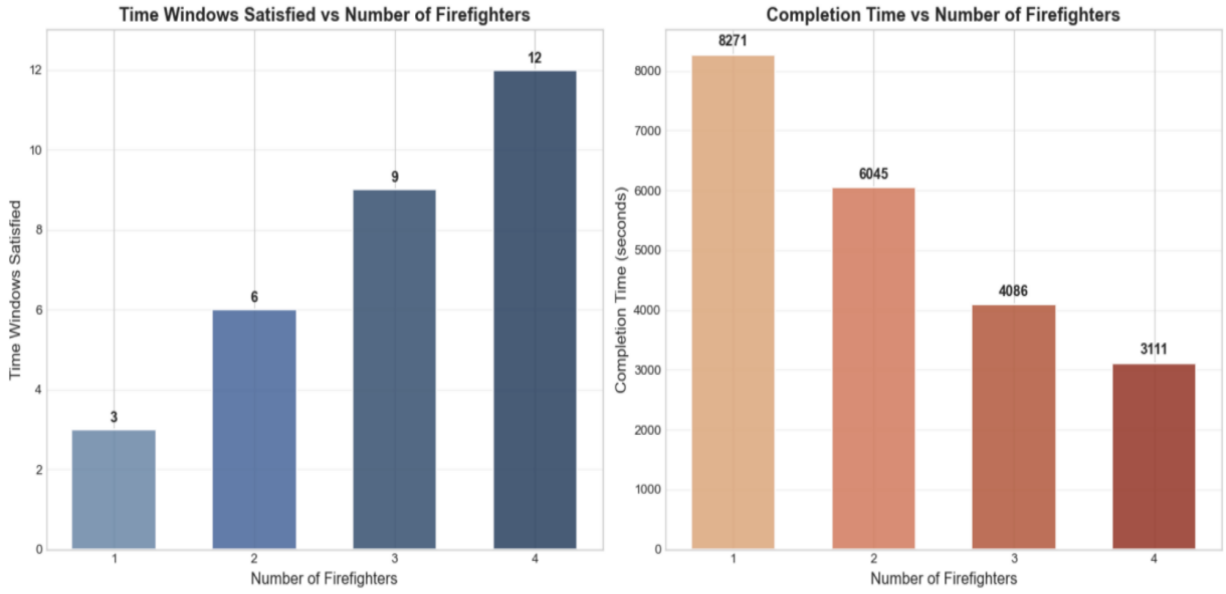


Figure 2. Change of time windows and completion time with different numbers of firefighters

7.4. Key Findings

Marginal benefit: Second firefighter provides largest time reduction (42% in Scenario 1); returns diminish beyond 3-4 responders.

Exit assignment: K-means clustering with same-entry-same-exit minimizes cross-building travel.

Dynamic risk: Fire spread forces larger teams (2 responders→ 4 responders in 7.3) to meet deadlines before rooms become inaccessible.

Priority integration: High-risk rooms (childcare, lab) consistently served first, meeting 900 s deadlines.

8. Model Evaluation of Strengths and Weaknesses

8.1. Model Strengths

Our framework demonstrates several key strengths:

Realistic Building Representation. The dual-approach graph abstraction captures real-world navigational constraints: Manhattan distance for open layouts, hallway intersection nodes with Floyd-Warshall shortest paths for complex corridor-based buildings.

Dynamic Risk Adaptation. The time-dependent risk model $\rho_i(t)$ with sigmoidal growth and room-specific critical times $t_{i,critical}$ realistically represents fire progression, capturing both immediate threat activation and flashover development.

Comprehensive Time Estimation. Travel and service times integrate multiple realistic factors including room-type sweep rates $\alpha(\tau_i)$(5–25 s/m²), emergency multipliers $\beta(\mathcal{E})$ (fire: 1.3, earthquake: 1.5), floor risk factors (increasing with height), door and stairwell transition times (10–15 s).

Safety-Centric Accessibility. Node accessibility $\psi i(t)$ progressively excludes hazardous areas once risk exceeds ρ_{safe} = 0.75, preventing life-threatening route assignments.

Multi-Emergency Flexibility. Parameter scaling with emergency-specific multipliers adapts the model to fires, gas leaks, earthquakes, and other scenarios.

Computational Efficiency. Floyd-Warshall precomputation enables constant-time distance lookups; k-means clustering reduces problem complexity while maintaining spatial efficiency.

8.2. Model Weaknesses

Lack of Response to Accident Our route plan is predetermined before the start of evacuation, and strongly relied on the execution of firefighters and accuracy of data. We didn't provide an alternative when firefighters might find different occupancy and emergency in actual scenario, and need to change their route.

Accuracy for Fire Propagation Current fire propagation modeling assumes independent fire development across rooms with constant flammability parameters. However, recent research demonstrates that probabilistic fire spread models incorporating room interconnectivity, fuel load distributions, and ventilation conditions provide more realistic hazard predictions for multi-compartment structures [4].

8.3. Sensitivity and Robustness Analysis

Table 4. Robustness Performance Under Extreme Conditions

Scenario	Performance Ratio	Avg. Time Change	Route Changes
Baseline	100.0%	0s	0
+20% Emergency Multiplier	108.3%	79s	1
-20% Emergency Multiplier	86.2%	-102s	2
+15% Service Time	127.4%	48s	0
-15% Service Time	92.5%	-23s	0
+30% Fire Spread Constant(k)	113.6%	86s	3

Using the case of double floor layout consist of 16 nodes, we randomized different parameters by ranges above to test the robustness of our model. The performance ratio is defined by (Actual Makespan of Perturbed Case / Optimal Makespan of Baseline Model)× 100%. This ratio directly indicates how much longer the operation takes under perturbed conditions compared to the ideal baseline. As seen, our route plan demonstrates robustness under the randomized change of emergency and service time. However, the speed of fire spread is finitely influential to our route (3 out of 16 nodes change) because the availbility of rooms are changed through time.

Another phenomenon that can be noticed is that as parameter randomly changed, the influence on makespan is much less than the variance of parameters. This portraits that even though the time may randomly changed in actual scenario, the completion of our route plan will not be greatly influenced.

9. Conclusion

9.1. Key Findings

Our research developed and validated a robust, graph-theoretic optimization framework for emergency responder routing in simple and complex building environments. We find that:

• **Quantitative Efficacy:** In the one-floor office validation scenario, the model determined that a 2-responder team was optimal, achieving a complete sweep in **1,477 seconds under normal conditions** and **1,894 seconds under emergency fire conditions**. This demonstrates the model's ability to balance efficiency with the realistic slowdowns caused by protective gear and hazardous conditions.

• **Strategic Route Optimization:** The integration of **k-means clustering for exit assignment** ensured spatial efficiency. The generated routes (e.g., Responder 1: Ew → R1 → R2 → R3 → Ew) minimized cross-building travel and redundant paths, directly reducing the mission makespan T_{max}.

• **Dynamic Risk Management:** The time-dependent risk propagation model pi(t) and node accessibility function ψi(t) functioned as critical safety mechanisms. They successfully prevented the assignment of responders to hazardous areas, with rooms being automatically excluded from routes once their risk exceeded the safety threshold (ρ_{safe} = 0.7).

• **Adaptability and Realism:** The model's flexibility was confirmed through its handling of diverse building layouts (open-plan vs. corridor-based) and emergency types (fire, gas leak, earthquake) through parameters.

9.2. Recommendations

Pre-Planning and Zoning. Pre-process floor plans into graph structures and use k-means clustering to divide buildings into response zones permanently assigned to specific exits, enabling immediate organized deployment.

Dynamic Risk-Based Prioritization. Prioritize rooms based on dynamic risk scores incorporating room type, occupancy, and floor level. High-risk rooms should be serviced early before becoming inaccessible.

Building-Specific Strategies.

Single-floor offices: Use Manhattan distance, balance loads across multiple exits.

Multi-story buildings: Use corridor-based shortest paths, assign responders to clusters.
High-risk facilities: Encode high-priority rooms (labs, daycare) with elevated risk scores and longer service times.

9.3. Future Work

Human Behavior Modeling. Integrate occupant behavior during evacuation (panic, exit familiarity, group dynamics) affecting corridor congestion and requiring dynamic travel time recalculation, .nabling more responsive routing strategies [8].

Communication Failure Robustness. Develop decentralized optimization algorithms that maintain effectiveness when central command communication is degraded or lost.

High-Fidelity Physical Simulation. Couple routing model with computational fluid dynamics (CFD) for physics-based fire and smoke propagation, replacing parametric risk estimates, significantly improving hazard assessment accuracy [9]

Multi-Objective Optimization. Expand from single objective minimize makespan to balance mission makespan, responder risk exposure, and potential occupant casualties, providing planners with Pareto optimal strategies.

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