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Optimization of multi-stage production inspection based on a dynamic decision tree

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Abstract. This study proposes an optimized production monitoring solution to address these challenges. By employing hypothesis testing, binomial distribution, and Poisson distribution, a sampling inspection method is designed to determine the minimum sample size and defect threshold for detecting defective products. Subsequently, a multi-stage dynamic decision tree model is developed to decompose the complex production process into multiple decision nodes, enabling the identification of the optimal decision sequence and improving process efficiency. Furthermore, considering more complex real-world production scenarios involving various processes and components, the concept of semi-finished products is introduced. A mixed-integer programming model is constructed, integrating defect rates and process performance, to evaluate the overall performance under different strategies. The results demonstrate that comprehensive inspection of components, avoidance of redundant inspections for semi-finished and finished products, and non-disassembly of non-conforming products can significantly reduce the inspection workload while ensuring quality and efficiency.

Keywords: decision optimization; hypothesis testing; dynamic decision tree models; mixed integer programming.

1. Introduction

Traditional quality inspection methods, such as full or empirical inspection, ensure product quality to some extent but suffer from high resource requirements and low efficiency, which have become bottlenecks for enterprise development. Full inspection requires every product to be tested, significantly increasing the inspection workload, especially in mass-production environments. Empirical inspection, on the other hand, relies on the subjective judgment and personal experience of operators, making it difficult to guarantee accuracy and consistency, which may result in unstable product quality. Therefore, it is essential to develop a method that effectively controls product quality while optimizing inspection processes to enhance the overall performance of manufacturing enterprises.

In this study, the primary goal is to propose an optimized production monitoring solution that significantly enhances manufacturing enterprises' process efficiency while ensuring product quality. The significance of this research lies in its potential to revolutionize production inspection practices by integrating modern statistical theories and decision science methods. The approach not only reduces inspection workload but also ensures product quality, thereby contributing to the sustainable development of manufacturing enterprises.

The innovation lies in the design of a sampling inspection plan based on hypothesis testing, binomial distribution, and Poisson distribution, By employing binomial and Poisson distributions to model the detection process of nonconforming products and designing sampling schemes at different confidence levels which enables the determination of the minimum sample size and defect threshold. Additionally, we introduce a multi-stage dynamic decision tree model that breaks down the complex production process into interconnected decision nodes. Decisions at each node influence the current production state and determine the possible paths and conditions for subsequent steps. By traversing all potential decision paths, the model identifies and selects the decision sequence that optimizes the overall process efficiency. The model considers defect rates for components, semi-finished products, and finished products. Through precise calculations and simulations, it provides enterprises with the optimal inspection strategy tailored to their specific production scenarios. This study provides a solid theoretical and practical foundation for the development of intelligent and automated production inspection systems. Furthermore, the rapid evolution of technology and increasing market demands for high-quality products underscore the urgency of optimizing production inspection processes. Research responds to this need by offering a systematic and efficient solution that addresses the challenges faced by modern manufacturing enterprises.

2. Sampling Inspection Plan Design

The data for this study was obtained from https://www.mcm.edu.cn/. A sampling and testing scheme for product parts is designed for a production supplier of a manufacturing company to determine whether the defective rate of spare parts exceeds the nominal value with a minimum number of tests. According to the concept of sampling distribution [1], if an experimenter carries out an experiment in which an infinite number of measurements are made, the frequency of occurrence of all measurements constitutes an overall distribution, which is a normal distribution [2]. In this paper, it is assumed that the nominal value of the production parts is 0.1, and a hypothesis testing scheme is used to help companies determine whether to accept the spare parts provided by the production supplier with the minimum number of tests.

In the real production situation, enterprises mainly face two kinds of production situations, the first is the sample detection when the defective rate is high, and the second is the sample detection when the defective rate is low. To detect the first situation, this paper assumes that the defective spare parts obey the binomial distribution, and takes the binomial test to calculate. For the second case of detection, this paper assumes that the spare parts defective obey the Poisson distribution, through the calculation of the defective threshold.

Hypothesis testing uses the ideas of the counterfactual method and the principle of small probability events to formulate the hypothesis H_0 and then make a decision to accept or reject H_0 based on the statistics of the sample [3].

First of all, assuming that the enterprise needs to detect a batch of spare parts defective rate is more than the nominal value, this paper through the hypothesis test achieves the:

The original hypothesis H_0: the defective rate of spare parts is less than or equal to the nominal value $p \leq p_0$.

Alternative hypothesis H_1: the defective rate of spare parts is greater than the nominal value $p > p_0$.

According to different confidence levels, design different sampling schemes:

Under a 95% confidence level, if the defective rate of spare parts is found to be more than the nominal value, the spare parts will be rejected. At a 90% confidence level, accept the spare parts if it is determined that the defect rate does not exceed the nominal value.

Assuming that in the case of a high defective rate, this paper uses binomial distribution to describe the detection process of nonconforming products; in the case of a low defective rate, this paper uses Poisson distribution to describe the detection process of nonconforming products.

2.1. Binomial Distribution

The binomial distribution is used to simulate the sampling and testing process of defective products. The binomial distribution, as a common discrete distribution, is widely used in many fields such as medical diagnosis, quality testing, and risk identification. In these practical applications, it is often necessary to give confidence intervals for the parameters of the binomial distribution, and finding the optimal confidence intervals is a long-standing research problem [4]. The binomial distribution is used to solve the problem with an error range E of 0.5, and the binomial distribution is used to simulate the sampling and detection process of substandard products: the number of spare parts taken from the sample is set to be n, and the number of substandard products is set to be X. The parameters of the binomial distribution are n and p. The parameters of the binomial distribution are set to be n and p. X~B(n,p).

Where nk is the number of combinations, denoting the probability of drawing k defective items from the nth spare part. According to the central limit theorem, when the sample size is large, the binomial distribution of the estimate of the defective rate can be approximated as a normal distribution $\hat{p} \sim N(p, \sqrt{\frac{p(1-p)}{n}})$, and the null hypothesis can be rejected by calculating the standardized test statistic Z, which can be compared with the critical value in the standard normal distribution table to decide whether to reject the null hypothesis:

$$Z = \frac{\hat{p}-p_0}{\epsilon} = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{1}$$

Where ϵ is the permissible error, p_0 is the nominal value, and $\hat{p}$ is the rate of substandard products in the sample.

By calculating the standardized test statistic Z, it is possible to decide whether to reject the null hypothesis by comparing it with the critical values in the standard normal distribution table.

①Calculate the minimum sample size

To control the testing C_{test} of the enterprise, and at the same time ensure the reliability of the test results, the sample size n needs to be optimized, which can be derived through the derivation of Equation (2):

$$n = \left(\frac{Z_{1-\alpha} \times \sqrt{p_0(1-p_0)}}{\epsilon}\right)^2 \tag{2}$$

Where $Z_{1-\alpha}$ is the critical value of normal distribution under different confidence intervals; p_0 is the nominal defective rate; ϵ is the permissible error, according to the previous production experience, this paper sets the permissible error as 0.05.

By checking the table, the critical value of normal distribution under different confidence intervals can be obtained, from which it can be calculated that the minimum sample size is 139 under 95% confidence level; under 90% confidence level, the minimum sample size is 98.

②Threshold calculation for the number of defective products

After determining the number of samples, the minimum value of the number of defective products detected within different decisions can be calculated, and the formula for calculating the threshold value of the number of defective products is as follows:

$$c = n \times p_0 - Z_{1-\alpha} \times \sqrt{n \times p_0 \times (1-p_0)} \tag{3}$$

Calculations can be made to derive thresholds for the number of defective parts at different confidence intervals, with the maximum number of defective parts rejected being 20 at a confidence level of 95, and the minimum number of defective parts received from this batch of spare parts being 14 at a confidence level of 90. results are shown in Fig 1 and Fig 2.

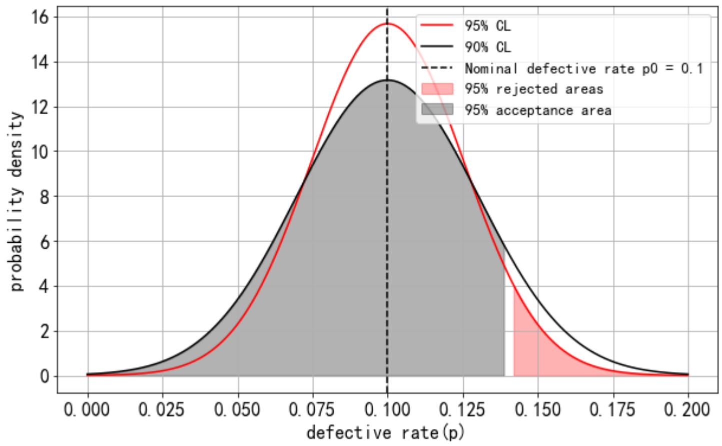

Figure 1. Example of a figure caption

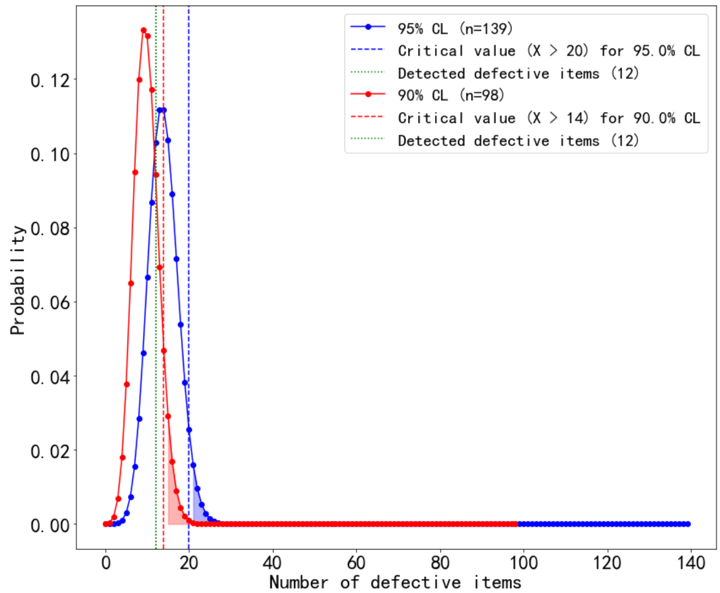

Figure 2. The normal distribution curve of defective rate under different confidence intervals

The blue area represents the area where, with 95% confidence, the spare parts were found to be defective more than the nominal value and the shipment was rejected; the red area represents the area where, with 90% confidence, the spare parts were found to be defective up to the nominal value and the shipment was accepted.

2.2. Poisson distribution

Consider further that, assuming a low rate of substandard products (less than 10%), a Poisson distribution is used to model the process of sampling and testing samples with a probability mass function.

The distributions of random variables have attracted a lot of interest. In probability theory and statistics, their probability density functions have been crucial. As a result, distributions have been extensively studied. Many other types of distributions, including the binomial distribution, the Poisson distribution, and the hypergeometric distribution, are taken into account from real-world scenarios. The probability density function of a Poisson distribution for a discrete random variable x is as follows [5]:

$$P(X=k)=\frac{e^{-\lambda}\lambda^{k}}{k!} \tag{4}$$

The distributions of random variables have attracted a lot of interest. In probability theory and statistics, their probability density functions have been crucial. As a result, distributions have been extensively studied. Many other types of distributions, including the binomial distribution, the Poisson distribution, and the hypergeometric distribution, are taken into account from real-world scenarios.

Where λ is the average number of occurrences in the unit space, i.e. the average defective rate; k is the number of observed defective products. As in the Poisson distribution, usually not directly calculate the minimum sample size, because the Poisson distribution assumes an infinite sample space, so this paper in the small sample size detection first assumed a 'reference sample size', set the production supplier to provide spare parts for 400 pieces. The Poisson distribution graph is shown in Fig3:

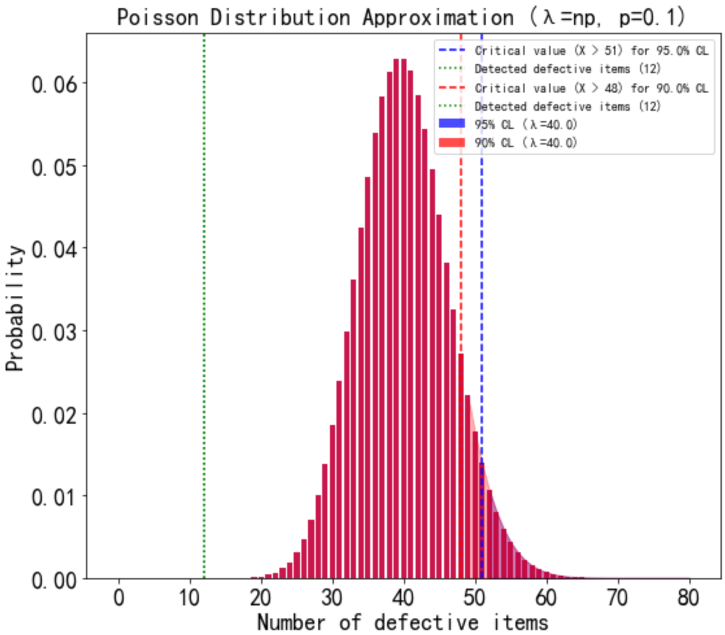

Figure 3. Poisson distribution graph

3. Cost-Benefit Optimization in Multi-Stage Dynamic Decisions

Consider further the decision-making allocation of the enterprise's production suppliers at the production stage, assuming that the enterprise has four stages of decision-making in the production process of the product: spare parts testing, finished product testing, dismantling of non-conforming products, and exchange of non-conforming products. If you choose to dismantle the nonconforming products, you need to repeat the parts testing and finished product testing; if you choose to recover the nonconforming products in the hands of customers, you need to repeat the process of dismantling the non-conforming products.

It is assumed that this paper needs to formulate a decision-making program for each stage of the production process of the enterprise, which involves the testing of spare parts and finished products, dismantling, and the handling of non-conforming products, etc., which can be regarded as a multi-stage decision-making problem. This question may arise because there are different decision-making methods and various studies on supplier selection problems [6]. The multi-stage design considers several decision periods for random variables. The main difference between the two-stage stochastic model is that the value of the decision variables can change at predetermined decision periods [7].

3.1. Constructing the objective function

To simulate the steps in the above production scenario, the paper gives the specifics of the production encountered as shown in TABLE I, TABLE II, and TBALE III:

Table 1. Situations encountered by enterprises in PRODUCTION (1)

Situations	Spare parts 1			Spare parts 2		
	p	unit	C_{test}	p	unit	C_{test}
1	10%	4	2	10%	18	3
2	20%	4	2	20%	18	3
3	10%	4	2	10%	18	3
4	20%	4	1	20%	18	1
5	10%	4	8	20%	18	1
6	5%	4	2	5%	18	3

Table 2. Situations encountered by enterprises in PRODUCTION (2)

Finished product			Unqualified products	
assembly	C_{test}	selling	replacement losses	dismantling
6	3	56	6	5
6	3	56	6	5
6	3	56	30	5
6	2	56	30	5
6	2	56	10	5
6	3	56	10	40

Firms need to calculate the total cost TC, total revenue TR, and finally total profit π based on elements such as C_{bought}, C_{test}, and defective rate.

3.2. Process Analysis

First of all, suppose the enterprise needs to purchase two different kinds of spare parts, namely, spare part 1 and spare part 2, and the purchase quantity of each kind of spare part is set as $N_{bought1}$ and $N_{bought2}$ respectively, and the corresponding unit price is $Price_1$ and $Price_2$.

For the inspection process of spare parts, the enterprise can choose whether to inspect spare parts 1 and 2 and define x_1 and x_2 as the decision variables of whether to inspect spare parts 1 and 2 respectively:

1)$x_1 = 1$ means test for Part 1.

2)$x_1 = 0$ means do not test for Part 1.

3)$x_2 = 1$ means test for Part 2.

4)$x_2 = 0$ means do not test Part 2.

After the operation on the parts is completed, a batch of finished products will be produced, and the number of finished products N_{total} depends on the number of qualified parts. Let parts 1 and parts 2 of the defective rate of p_1 and p_2, if the enterprise chooses to detect certain spare parts, then all the defective will be excluded; if not, then the number of qualified parts will be affected by its defective rate:

1) If detecting spare parts 1 (i.e. x_1=1), the number of qualified spare parts is $N_{qualified1}$;

2) If part 1 is not detected (i.e. x_1=0), the number of qualified parts is $N_{qualified1}$·(1-p_1);

3) Similarly, for part 2, if x_2=1, the number of qualified parts is $N_{qualified2}$; if x_2=0, the number of qualified parts is $N_{qualified2} \cdot (1 - p_2)$.

Therefore, the number of finished products N_{total} is determined by the minimum value of qualified parts and parts2, i.e.:

$$N_{total} = \min(x_1 \cdot N_{qualified1} + (1 - x_1) \cdot N_{qualified1} \cdot (1 - p_1), x_2 \cdot N_{qualified2} + (1 - x_2) \cdot N_{qualified2} \cdot (1 - p_2)) \tag{5}$$

Since different operational decisions produce different finished products, the finished product defective rate p_{finish} is solved as follows:

1)If both spare part 1 and spare part 2 are tested (x_1=1 and x_2=1), the finished product defective rate is 1-$p_1 p_2$;

2) If only part 1 is tested (i.e., x_1=1 and x_2=0), the finished product defect rate is p_2;

3) If only part 2 is tested (i.e., x_1=0 and x_2=1), the finished product defect rate is p_1;

4) If both parts are not tested (x_1=0 and x_2=0), the defective rate of the finished product is the joint probability of the defective rate of the two parts, i.e.: $p_1 + p_2 - p_1 \cdot p_2$.

In summary, the finished product defective rate p_{finish} can be uniformly expressed as:

$$p_{finish} = 1 - p_1 p_2 + (1 - x_1) \cdot p_1 + (1 - x_2) \cdot p_2 - (1 - x_1) \cdot (1 - x_2) \cdot p_1 \cdot p_2 \tag{6}$$

For the finished product after production, the enterprise can decide whether to test the finished product or not, and the decision of testing the finished product is indicated by x_{finish}, where x_{finish}=1 means testing the finished product; x_{finish}=0 means not testing the finished product.

Meanwhile, for the unqualified finished products, the enterprise can choose to dismantle or discard them.

①Calculation of TC

For the six cases in the question, the TC to be calculated in this paper includes the C_{bought}, C_{test0}.

C_{bought} can be expressed as:

$$C_{bought} = N_{bought1} \cdot Price_1 + N_{bought2} \cdot Price_2 \tag{7}$$

C_{bought} is the basis that firms pay to obtain spare parts. For spare parts, C_{test0} can be expressed as:

$$C_{\text{test0}} = x_1 \cdot N_{bought1} \cdot Test_1 + x_2 \cdot N_{bought2} \cdot Test_2 \tag{8}$$

The TC includes C_{bought}, C_{test0}, $C_{\text{disassemble}}$ of non-conforming finished product, and C_{deal} of non-conforming finished product. Based on the various equation terms that have been defined earlier, the final can be derived:

$$TC = (N_{bought1} \cdot Price_1 + N_{bought2} \cdot Price_2) + (x_1 \cdot N_{bought1} \cdot Test_1 + x_2 \cdot N_{bought2} \cdot Test_2 +) + (p_{finish} \cdot N_{total} \cdot Test_{finish}) + (x_{dis} \cdot p_{finish} \cdot N_{finish} \cdot r) + ((1 - x_{dis}) \cdot p_{finish} \cdot N_{finish} \cdot l) \tag{9}$$

②Calculation of TR

TR comes from the sale of qualified finished goods, the quantity of qualified finished goods is $N_{total}\cdot$ (1-p_{finish}), and the selling price of each piece of finished goods is S. The TR can be expressed as:

$$TR = N_{total}\cdot(1\text{-}p_{finish})\cdot S \tag{10}$$

③Calculation of π

The total profit of the enterprise is the TR minus the TC, based on the derivation of the above formula, the complete profit formula expression is:

$$\begin{gathered} \pi = N_{total} \cdot (1 - p_{finish}) \cdot S - \\ (N_{bought1} \cdot Price_1 + N_{bought2} \cdot Price_2) + \\ (x_1 \cdot N_{bought1} \cdot Test_1 + x_2 \cdot N_{bought2} \cdot Test_2 +) + \\ (x_{finish} \cdot N_{total} \cdot Test_{finish}) + \\ (x_{dis} \cdot p_{finish} \cdot N_{finish} \cdot r) + \\ ((1 - x_{dis}) \cdot p_{finish} \cdot N_{finish} \cdot l) \end{gathered} \tag{11}$$

3.3. Multi-stage dynamic decision tree modeling

A decision tree may be perceived as a method of dissecting data sets into more minute subsets with an increase in the depth of a hypothetical tree. A tree usually comprises 'decision nodes' and 'leaf nodes' where numerous branches may fall on each decision node [8].

Based on the above analysis, this paper innovatively constructs a multi-stage dynamic decision tree model, the core strategy of which is to refine the complex production process into a series of closely connected decision nodes, where decisions at each node not only immediately affect the current state, but also profoundly shape the alternative paths and conditions of the subsequent steps. This paper can systematically traverse all potential decision trajectories, and then select the set of decision sequences that optimize the overall profitability of the firm.

①Decision variables

According to the actual production scenario simulated in this paper, two main decision variables are defined, namely:

x_1: whether or not to detect the spare parts 1, 1 means to detect, 0 means not to detect.

x_2: whether or not to test spare part 2.

②State Variables

State variables are used to describe the decision-making state of the model at each stage, in this paper it sets the TC of the current stage as $C_{current}$ and the expected TR $as\ R_{current}$.

③Objective function

$$Max_{x_1,x_2}(TR - TC) \tag{12}$$

④Constraints

$$x_1, x_2 \in \{0,1\}, TR, TC \geq 0 \tag{13}$$

⑤Optimising functions

$$Optimize(stage, C_{current}, R_{current}) = max_{x_{stage}}\{optimize\left(\begin{matrix} stage + 1, \\ C'_{current}, R'_{current} \end{matrix}\right)\} \quad (14)$$

Based on these components, the recursive optimization function, this paper can construct a decision tree model for enterprise suppliers in production decision-making. Each node of the decision tree represents a decision point, and each path from the root node to the leaf nodes represents a possible decision sequence. The optimal decision path is identified by comparing the final total benefits of different paths. The results of the visualization of different decision paths are shown in Fig.4. to Fig.6.:

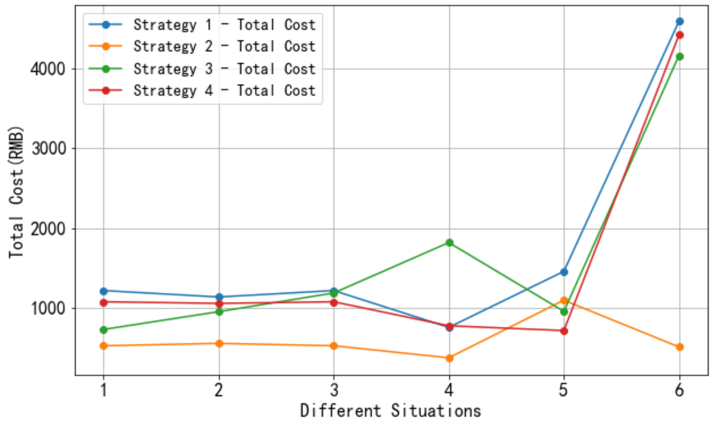

Figure 4. Line graph of TC under different strategies

Figure 5. Line graph of TR under different strategies

Figure 6. Line graph of π under different strategies

The above visualizations are summarised in TABLE III

Table 3. Optimal decision for each case

Situations	Strategy	1	2	Finished product	Defective product
1	1	detected	detected	detected	dismantle
1	2	detected	not	detected	not
1	3	not	not	not	dismantle
1	4	not	detected	detected	dismantle
2	1	detected	detected	detected	dismantle
2	2	detected	not	detected	not
2	3	not	not	not	dismantle
2	4	not	detected	detected	dismantle
3	1	detected	detected	detected	dismantle
3	2	detected	not	detected	not
3	3	not	not	not	dismantle
3	4	not	detected	detected	dismantle
4	1	detected	detected	detected	dismantle
4	2	detected	not	detected	not
4	3	not	not	not	dismantle
4	4	not	detected	detected	dismantle
5	1	detected	detected	detected	dismantle
5	2	detected	not	detected	not
5	3	not	not	not	dismantle
5	4	not	detected	detected	dismantle
6	1	detected	detected	detected	dismantle
6	2	detected	not	detected	not
6	3	not	not	not	dismantle
6	4	not	detected	detected	dismantle

Combining the six scenarios, it can be obtained that the overall π of \$4539.0 can be obtained by adopting Strategy 2 in the face of Scenario 6.

Inspection of Spare Part 1: The enterprise chooses to inspect Spare Part 1, although it will incur additional inspection parts, due to the relatively high defective rate of Spare Part 1, if it is not inspected, more defective products may flow into the finished product stage, which in turn affects the defective rate of the finished product. Therefore, most of the unqualified spare parts can be eliminated by testing spare parts 1.

Non-detection of spare parts 2: On the basis of detecting spare parts 1, non-detection of spare parts 2 can reduce the value of detection on the basis of affecting the defective rate of finished products as little as possible.

Finished product testing: under the condition of only detecting parts 1, the finished product can be tested after the production of the finished product to eliminate most of the unqualified finished products, greatly reducing the probability of non-conforming products into the market, thereby reducing the value of non-conforming products replacement.

Disassembly of unqualified finished products: by disassembling unqualified finished products, at least one qualified spare part can be obtained and put into production again without affecting the quality of the finished product, which can minimize the production value.

4. Multi-Stage Inspection and Optimization Decision Model in Complex Production Processes

Considering the existence of substandard rates at all stages of the production process, the production process containing two processes and eight spare parts, for example, needs to be solved for the production of multiple processes and spare parts. For this purpose, a specific decision-making solution is required, and the corresponding basis and indicator results are provided. This paper aims to provide a decision-making solution for a complex production process involving multiple processes and multiple parts, where the defective rate of parts, semi-finished products, and finished products is known. Due to the reality that the production process is more complex, including multiple processes (specifically m processes) and multiple spare parts (specifically n spare parts), this paper to 2 processes and 8 spare parts as a representative of the case, the production of spare parts for enterprises to make a more detailed analysis of the decision-making process.

4.1. Multi-stage decision problem

The model of the multi-stage decision-making system considers social, environmental, technical, organizational, and managerial aspects of macro, micro, and mezzo environments [9]. In the multi-stage production process, due to the existence of multiple processes and the influence of multiple spare parts, the decision of each process may have a certain impact on spare parts or finished products. In addition, considering the multi-stage decision-making in the production process may have an impact on the defective rate, this paper also introduces the concept of semi-finished products, the defective rate of semi-finished products will also have an impact on the quality of the subsequent process.

Therefore, in the multi-stage decision-making problem of the production process, this paper needs to consider the following factors:

1) whether to test the spare parts, semi-finished products, and finished products.

2) Based on the test results of the spare parts and semi-finished products, the decision to replace or dismantle defective spare parts or semi-finished products.

3) Calculation of the TC and the expected total profit under different strategies, to select the optimal decision-making scheme.

①Decision Variables:

To make the optimal decision, the following decision variables are defined in this paper:

x_i: whether to inspect part i (i=1,2,......,n) or not, taking the value of 0 or 1. If $x_i = 1$, it means that part i is inspected.

y_j: whether to detect semi-finished product j(j=1,2,...,m) of process j, the value is 0 or 1. If $y_j = 1$, it means that semi-finished product j is detected.

z: whether to detect the finished product, takes the value of 0 or 1. If z=1 means to detect the final product.

d: whether to dismantle the unqualified finished product, the value is 0 or 1. If d=1, it means to dismantle the unqualified finished product, otherwise scrap.

②p_{half}

The defective rate of the semi-finished product is determined by the defective rate of the spare parts composing the semi-finished product. Assuming that semi-finished product j consists of spare parts i, the defective rate p_{half} of semi-finished product j can be expressed as:

$$p_{half} = 1 - \prod_i (1 - p_i) \tag{15}$$

This means that if the pass rate of spare part i is $1 - p_i$ the defective rate of the process is determined by the combined defective rate of all the spare parts involved in the process.

③Finished product defective rate

The defective rate of the finished product, p_{finish}, is determined by the propagation of the defective rate of each process, p_{half}, and the defective rate of the final product can be expressed as follows:

$$p_{finish} = 1 - \prod_{j=1}^{m} (1 - p_{half}) \tag{16}$$

The qualified quantity of the final product is:

$$N_{finish}^{\text{qualified}} = N_{finish} \cdot (1 - p_{finish}) \tag{17}$$

④C_{test}

The C_{test} includes the testing spare parts, semi-finished products, and finished products, where the testing spare parts are:

$$C_{\text{test0}} = \sum_{i=1}^{n} N_i \cdot C_{testi} \cdot x_i \tag{18}$$

Where N_i is the number of spare parts i purchased; C_{testi} is the testing spare parts i; and x_i is the decision variable of whether to test spare parts i or not.

The semi-finished products are:

$$C_{\text{test1}} = \sum_{j=1}^{m} N_j \cdot C_{halfj} \cdot y_j \tag{19}$$

Where C_{halfj} is the semi-finished product j; y_j is the decision variable for whether to test semi-finished product j or not.

The detection of the finished product is:

$$C_{\text{test2}} = N_{finish} \cdot C_{finish} \cdot z \tag{20}$$

Where C_{finish} is the test of the finished product; z is the decision variable of whether to test the finished product or not.

⑤C_{deal}

For the finished product that fails to pass the test, there exist two ways to deal with it, which are dismantling or scrapping. Among them, the $C_{\text{disassemble}}$ is solved as:

$$C_{\text{disassemble}} = d \cdot p_{finish} \cdot N_{finish} \cdot C_{\text{r}} \tag{21}$$

Where C_{r} is the $C_{\text{disassemble}}$ of the finished product. If dismantling is selected, $C_{\text{disassemble}}$ is incurred. If scrap is selected, C_{scrap} is incurred, and C_{scrap} is solved as:

$$C_{scrap} = (1 - d) \cdot p_f \cdot N_f \cdot C_{\text{l}} \tag{22}$$

Where C_{l} is the interchange loss of the finished product. Thus, C_{deal} can be expressed as:

$$C_{\text{deal}} = (d \cdot p_{finish} \cdot N_{finish} \cdot C_{\text{r}}) + [(1 - d) \cdot p_f \cdot N_f \cdot C_{\text{l}}] \tag{23}$$

⑥TR

The TR is expressed as:

$$TR = N_{finish}^{\text{qualified}} \cdot S_{finish} \tag{24}$$

where S_{finish} is the market selling price of the finished product.

4.2. Mixed-Integer Programming Model

In this paper, the optimization problem of a production process is transformed into a mathematical model of mixed integer programming (MIP). In this model, the linear relationship runs through the objective function and the constraints, which ensures the simplicity and consistency of the problem solution [10]. At the same time, the concept of integer variables is introduced to reflect the integer constraints that must be followed for certain decisions in real production, such as production volume, number of equipment, etc., while the rest of the variables are kept flexible and allowed to take values in the real range.

Through this MIP modeling approach, this paper successfully transforms a complex production optimization task into a clearly structured and logically rigorous mathematical problem, making the solution process more systematic and scientific. With the help of efficient solution algorithms, the potential results of different decision-making scenarios can be comprehensively explored, and the optimal decision-making combinations that can maximize the economic benefits of the enterprise can be filtered out. This process not only improves the efficiency and accuracy of production decision-making but also provides powerful decision support for the sustainable development of enterprises.

①Objective Function

This paper takes maximizing profit as the objective function, it can be derived as:

$$\pi = TR - TC \tag{25}$$

The decision variables x_i, y_j, z, and d need to be optimized to achieve the solution to the objective function.

②Constraints

To guarantee the rationality of the production process, the following constraints are introduced in this paper:

Parts and semi-finished product inspection order constraint: semi-finished product j can only be inspected after all the parts and accessories that make up the semi-finished product have been inspected. The constraint is expressed as:

$$y_j \leq min(x_i), \forall i \in \text{Process j related spare parts} \tag{26}$$

Semi-finished and finished product inspection order constraint: the inspection of finished product can only be performed after all semi-finished products have been inspected with the constraint:

$$z \leq min(y_j)\ \forall j \tag{27}$$

4.3. Analysis of results

After the model is built, this paper firstly exhausts all the decision-making results and visualizes the results of the solution as Fig.7.:

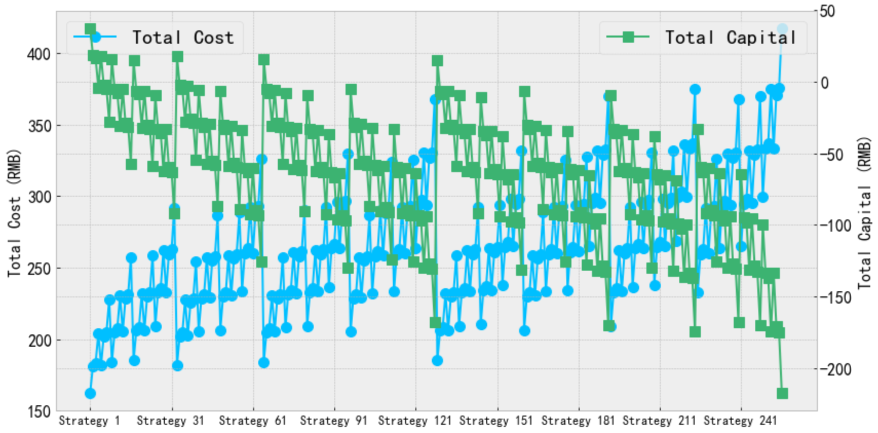

Figure 7. TC and π under different strategies

In this paper, it is possible to observe the relationship between TC and π under different strategies. The blue line in the graph represents the TC and the green line represents the π. The overall trend is clear, with the change of strategy status, the TC shows a gradual increase, while the π fluctuates more. To show the relationship between TC and π more intuitively, this paper draws a heat map between TC and π, as shown in Fig .8.:

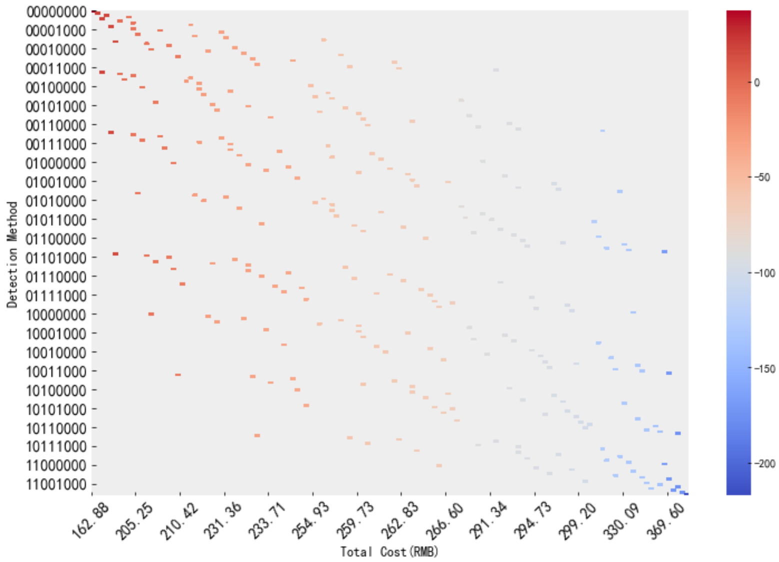

Figure 8. Heat map between TC and π

The more concentrated and more profitable areas of the heat map are centered on lower expected values (bottom left), and these combinations are usually strategies that have not been subjected to too much testing. This part of the strategy shows better economics, suggesting that moderate detection can better control TC and π.

Ultimately, the optimal decision made by the mixed integer programming model is [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0,0,0,0,0,0],i.e.:

For spare parts, 1-8: all tested, through this decision can ensure that through the detection of each spare part are qualified products, there are no defective products, thus improving the efficiency of the subsequent process. For semi-finished products 1-3: all do not detect; in the detection of spare parts 1-8 based on this decision can be guaranteed based on semi-finished products pass rate greatly reduces C_{test}, for the enterprise to greatly reduce TC. For the finished product: no testing, due to all the spare parts for testing, to ensure that the qualified rate of the finished product, not test the finished product to help minimize TC. For unqualified finished products: no disassembly, under the condition of a high qualified rate of finished products, disassembling unqualified finished products and assembling them again is more costly than disassembling them, to minimize the TC, no disassembly of unqualified finished products is carried out.

By interpreting the results, this paper shows that all spare parts are tested for decision-making, all semi-finished products are not tested for decision-making, all finished products are not tested for decision-making, and all unqualified finished products are not disassembled for decision-making. The above decisions greatly reduce C_{test} during the production of the product and contribute to profit maximization.

5. Conclusion

This study employs hypothesis testing to design a sampling program tailored to different confidence levels. For scenarios with high defect rates, a binomial distribution model is utilized, and at a 95% confidence level, a batch is rejected if the defect rate exceeds the threshold with a minimum sample size of 139. For low defect rates, a Poisson distribution is applied, and at a 95% confidence level, a batch is rejected if 51 defective products are found. The study then constructs a dynamic decision tree model to analyze a four-stage production process, developing optimal inspection strategies for each scenario. Across various scenarios, different inspection and handling approaches significantly affect production efficiency and process performance, with adjustments leading to substantial improvements, up to $3,900 under specific conditions. For complex production processes, the model is optimized by recalculating key metrics at each stage, formulating the solution as a mixed-integer programming problem. The optimal strategy involves comprehensive inspection of specific parts, omitting inspection for semi-finished and finished products, and no disassembly of nonconforming items.

As technology advances and markets evolve, future research directions can be further expanded. For instance, the application of artificial intelligence and big data in production inspection can be explored to enhance the intelligence and automation of decision-making. Additionally, more types of production processes and more complex decision-making scenarios can be studied to further improve and optimize production inspection solutions.

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