

Multi-Stage Defect Rate Optimization Using a Hybrid Model of Simulated Annealing and Monte Carlo Simulation

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Abstract. Amid global value chain restructuring and Industry 4.0 transformation, manufacturers face significant challenges in controlling defect rates across multi-stage production processes. This study develops a hybrid optimization model combining Simulated Annealing and Monte Carlo Simulation to balance quality control and cost efficiency. The framework incorporates a comprehensive cost structure covering components, semi-finished/finished products, and defect-related losses, while introducing a dynamic defect rate adjustment coefficient to quantify inspection-skipping risks at different stages. The model employs Simulated Annealing to optimize inspection strategies, with Monte Carlo methods verifying solution robustness. Sampling inspection techniques are integrated to enhance defect detection efficiency. Experimental results demonstrate that the optimized strategy reduces total production costs substantially, with the best-performing solution achieving a unit cost of 145 CNY while maintaining defect rates below 2.25%. Compared to conventional approaches, the new model cuts unnecessary inspection costs by approximately 14% without compromising quality standards. This research provides manufacturing enterprises, particularly in complex sectors like electronics, with a practical decision-making tool for dynamic production environments. The framework's adaptability supports supply chain quality management in the era of smart manufacturing, offering both methodological innovation and operational value for industry applications.

Keywords: Multi-stage inspection strategy; Defect rate adjustment; Simulated annealing; Monte Carlo simulation; Quality control optimization.

1. Introduction

Decades ago, the global economy entered the "global value chain era" [1]. Pol Antras and Davin Chor [2] attributed this transformation to the information technology revolution, whose generational advances (e.g., Internet, cloud computing) reduced cross-border coordination costs, enabling global dispersion of production. Enterprises relied more on foreign components, and intermediate product producers exported globally, tightening production links among nations [3].

Soon after, the Industry 4.0 technological revolution emerged, leveraging IoT, cloud computing, etc., to drive automation and data exchange in manufacturing, encompassing cyber-physical systems and smart factories [4]. This digital wave reshapes global resource allocation and competition, creating opportunities for developing countries to upgrade via digital transformation and for SMEs to participate in international trade. Decentralized IoT infrastructure reduces production/distribution costs, eliminating transaction costs to enable near-zero-cost sharing, while international competition forces manufacturers to optimize technologies and processes, enhancing supply chain efficiency [5]. Thus, trade structure transformation and technological revolution interact to fuel global corporate competition.

In this competitive market, controlling defect rates in multi-process production is key for manufacturers to cut costs and improve efficiency. For example, in electronics, component defects

affect yield, and undetected quality issues destabilize supply chains and damage brands. Controllable quality maintains reputation and reduces recall/legal risks [6].

Traditional detection methods have drawbacks: time-consuming, destructive testing (rendering samples unsalable), and sampling inspection risking missed defects [7]. Hence, optimizing multi-stage detection and building dynamic defect rate models with intelligent algorithms and big data are critical. Machine learning and deep learning enable real-time data-driven adjustment mechanisms. Tian Youwen [8] and Xue Yong [9] applied deep learning to fruit and apple defect detection with notable results. This approach dynamically determines inspection points and sampling rates to monitor quality and reduce costs. For instance, deep learning-based automotive surface inspection achieves 95% accuracy (10% higher than traditional methods), enhancing automation [10]. Dynamic defect rate models predict quality fluctuations to support production adjustments.

In decision-making, Tang Leceng [11] proposed a joint optimization model for order selection and equipment maintenance based on customer satisfaction, balancing efficiency, quality, and service. Liang Xiaoyu and Lu Zhiqiang [12] modeled equipment degradation via the Wiener process, proposing maintenance/replacement strategies to minimize costs. Multi-objective optimization is now essential for complex enterprise decisions in Industry 4.0.

Regarding algorithms, Wang Shiyong [13] used Monte Carlo and simulated annealing to address machine tool degradation and energy consumption, while Xiao Tianfei [14] applied genetic algorithms to injection molding machines, enhancing global search and precision control. Appropriate algorithms solve production issues, improving efficiency and reducing costs.

In summary, against the backdrop of global value chain restructuring and Industry 4.0, integrating trade structure transformation with intelligent manufacturing via optimized inspection strategies and algorithm models boosts efficiency, stabilizes supply chains, and safeguards brand reputation, ensuring corporate competitiveness in the global market.

2. Methodology

2.1. Optimization Model for Multi-Stage Defective Rate Control

To address the issue of defective rate control in multi-stage, multi-component production processes, this research proposes a multi-stage decision-making model that aims to minimize the total production cost while ensuring optimal quality control. The total cost is divided into four parts: component cost, semi-finished product cost, finished product cost, and loss cost. Moreover, this research introduces an adjustment coefficient for defective rates, quantifying the risks of not conducting inspections at different stages.

The optimization process is carried out using Simulated Annealing (SA) to determine the optimal inspection and defective product handling strategies. To validate the model's effectiveness, this research further employs Monte Carlo simulation, Genetic Algorithm (GA), and Particle Swarm Optimization (PSO) for comparative verification.

2.1.1. Mathematical Model

Let p_i denote the defective rate of component i , and α_i be the adjustment coefficient for non-inspected parts. If a part is not inspected, its defective rate increases by a factor of α_i . The defective rate of semi-finished and finished products is calculated recursively:

$$p_{semi} = 1 - \prod_{i=1}^n (1 - \alpha_i p_i) \quad (1)$$

$$p_{final} = 1 - \prod_{j=1}^m (1 - \alpha_j p_{semi,j}) \quad (2)$$

where n and m are the numbers of components and semi-finished products, respectively.

The total cost function consists of:

- (1) Inspection cost C_d at different production stages.
- (2) Assembly cost C_a , which accumulates through different process steps.
- (3) Defective handling cost, including scrap cost C_s and rework cost C_r .
- (4) Loss cost C_l , derived from market returns and quality penalties.

$$C_{total} = C_d + C_a + C_s + C_r + C_l \quad (3)$$

The optimization problem is formulated as:

$$\min C_{total} \text{ subject to } 0 \leq \tau, d_i \in \{0,1\} \quad (4)$$

where d_i is the binary decision variable indicating whether inspection is performed at stage i , and τ is the acceptable defective rate threshold.

2.1.2. Solution Method

The Simulated Annealing (SA) algorithm is applied to iteratively search for the optimal decision strategy by adjusting inspection policies at each stage. The temperature decay follows an exponential schedule:

$$T_k = T_0 \cdot \lambda^k \quad (5)$$

where λ is the cooling rate. The objective function is evaluated for different decision combinations, and Monte Carlo simulation is used to verify the robustness of the optimal strategy.

2.2. Optimization Model for Sampling Inspection

2.2.1. Solution Approach

In this stage, this research revisits the production decision strategies by introducing sampling inspection methods. In the previous stages, the defective rates for components, semi-finished products, and finished products were considered fixed. However, in this case, defective rates are estimated through sampling inspection, which introduces uncertainty. Therefore, this research corrects the defective rates using hypothesis testing and confidence intervals.

2.2.2. Mathematical Model for Sampling Inspection

This research introduces a sampling inspection model based on statistical hypothesis testing and confidence intervals. The defective rate is corrected using the following formula:

$$p' = p \pm Z_\alpha \sqrt{\frac{p(1-p)}{n}} \quad (6)$$

where p' is the corrected defective rate, p is the initial estimate of the defective rate, Z_α is the corresponding quantile of the standard normal distribution for the confidence level, and n is the sample size.

2.3. Cost Function and Total Cost Calculation

This research updates the cost calculation model to incorporate the uncertainty from the sampling inspection. The total cost function is given by:

$$C'_{total} = C_d + C_a + (C_s + C_r) \cdot p' + C_l \cdot p' \quad (7)$$

2.3.1. Solution Method for Sampling Inspection

This research apply enumeration to explore all possible detection strategies and adjust the cost model accordingly. The steps are as follows:

- (1) Adjust defective rates using hypothesis testing and confidence intervals based on sampling results.
- (2) Update the cost model to reflect the new defective rates.
- (3) Search for the optimal strategy using enumeration.

3. Result and discussion

During the solution process, this research utilized the enumeration method to comprehensively traverse all feasible inspection strategies and precisely calculated the corresponding costs using the adjusted total cost formula. By systematically comparing the total costs of different inspection strategy combinations, this research successfully identified the optimal inspection strategy with the lowest total cost.

- (1) Step 1: Based on the sampling inspection results, this research employed hypothesis testing and confidence interval estimation to calibrate the defect rates at each stage.
- (2) Step 2: This research refined the cost model and recalculated the inspection costs, assembly costs, disassembly costs, and expected loss costs due to defects at each stage. It is noteworthy that variations in the defect rate directly influence the losses from non-inspection or non-conforming product handling.
- (3) Step 3: This research again applied the enumeration method to search for the optimal decision combination across different production stages. This approach aimed to minimize the total cost while achieving the best balance between inspection accuracy and inspection cost.
- (4) Step 4: To aid decision-making, this research conducted a visual comparison of the total costs at each production stage, clearly showing the fluctuations in total costs under different inspection strategies. Additionally, this research performed a visual analysis of the results before and after the introduction of sampling inspection to intuitively illustrate the impact of strategy adjustments.

Following the introduction of the sampling inspection method, the total cost distribution under different strategy combinations exhibited the following characteristics:

(1). Total Cost Distribution Range:

The total cost for most strategy combinations was concentrated between 155 and 170 yuan, indicating relatively stable costs with minimal fluctuations under these strategies.

(2). Optimal Strategy:

This research used the simulated annealing algorithm to traverse all 8192 strategy combinations, evaluated the cost value of each strategy, and finally selected the strategy with the highest profit as the optimal strategy. The result is shown as Table 1.

Table 1. Total costs under various strategies

Strategy	Spare Parts Inspection Decision	Semi - finished Product Inspection Decision	Finished Product Inspection Decision	Total Cost (Yuan)
1	(T, T, T, T, T, T, T)	(T, T, T)	(T, T)	182.237127
2	(T, T, T, T, T, T, T)	(T, T, T)	(T, F)	172.237127
3	(T, T, T, T, T, T, T)	(T, T, T)	(F, T)	176.237127
4	(T, T, T, T, T, T, T)	(T, T, T)	(F, F)	166.237127
5	(T, T, T, T, T, T, T)	(T, T, F)	(T, T)	178.1256237
6	(T, T, T, T, T, T, T)	(T, T, F)	(T, F)	168.1256237
7	(T, T, T, T, T, T, T)	(T, T, F)	(F, T)	172.1256237
.....
8192	(F, F, F, F, F, F, F)	(F, F, F)	(F, F)	145.3735237

According to the calculation results, when comprehensively considering the cost and the defective rate of finished products, the optimal strategy combination is the 8192nd strategy group, with a total cost of approximately 147.62 yuan. If the defective rate of finished products is not considered, the total cost is approximately 145.37 yuan. And both point to the same decision (the 8192nd group): The adjusted defective rates are as follows: 0.15 for spare parts, 0.15 for semi-finished products, and 0.0225 for finished products. Based on the inspection decisions, no inspections will be conducted at any stage of the production process — including spare parts (F, F, F, F, F, F, F), semi-finished products (F, F, F), and finished products (F, F).

The defective rate of this strategy after adjustment is 2.25%, which is the optimal decision considering the total cost and the defective rate at present. After obtaining the final decision, this research use the Monte Carlo simulation algorithm, genetic algorithm, and particle swarm optimization algorithm for comparative analysis. When the defective rate of finished products is not considered, the calculated costs are all the same as the results obtained by the simulated annealing algorithm.

Strategy Combination 8 achieved the lowest total cost of approximately at an adjusted defect rate of 2.25%. This strategy effectively controlled production costs while ensuring product quality.

(3). Cost Optimization Effect:

With the sampling inspection method, companies can more accurately estimate defect rates at each stage, thereby optimizing production decisions and significantly reducing total costs.

When comparing the strategy cost differences before and after the introduction of sampling inspection, the following observations were made:

(1). Cost Reduction:

The implementation of sampling inspection led to a reduction in total costs across all strategy combinations, demonstrating the method's effectiveness in cutting unnecessary inspection costs while maintaining high-level quality control.

(2). Notable Cost Optimization:

The cost optimization effect was particularly significant for Strategy Combination 8, as shown in Table1 and Figure 1 and Figure 2, further confirming the effectiveness of the sampling inspection method in reducing production costs.

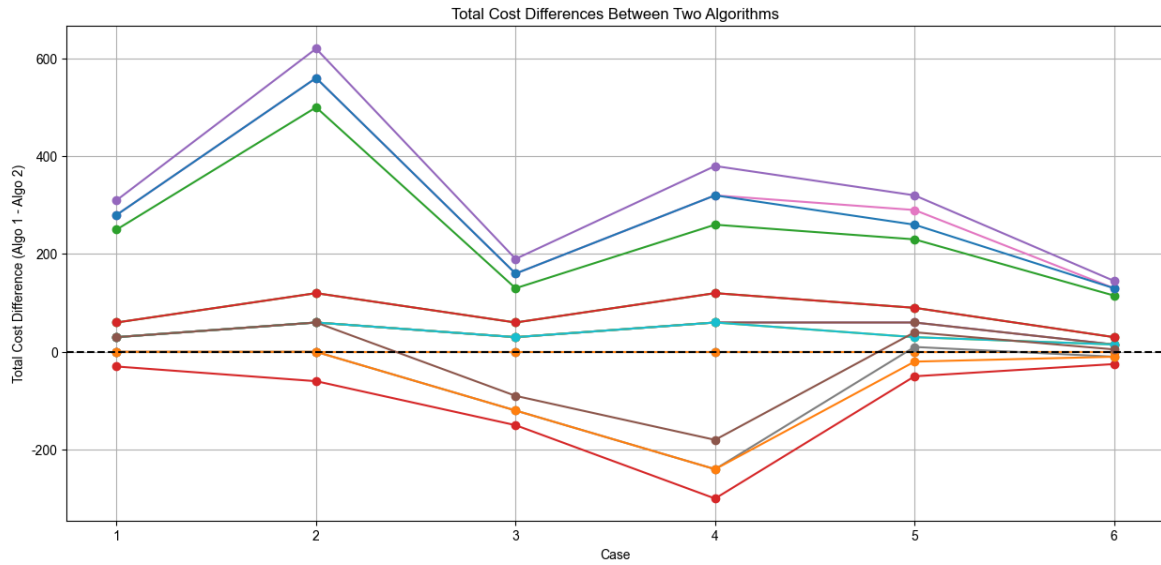


Figure 1. Total Cost Differences Between Two Algorithms

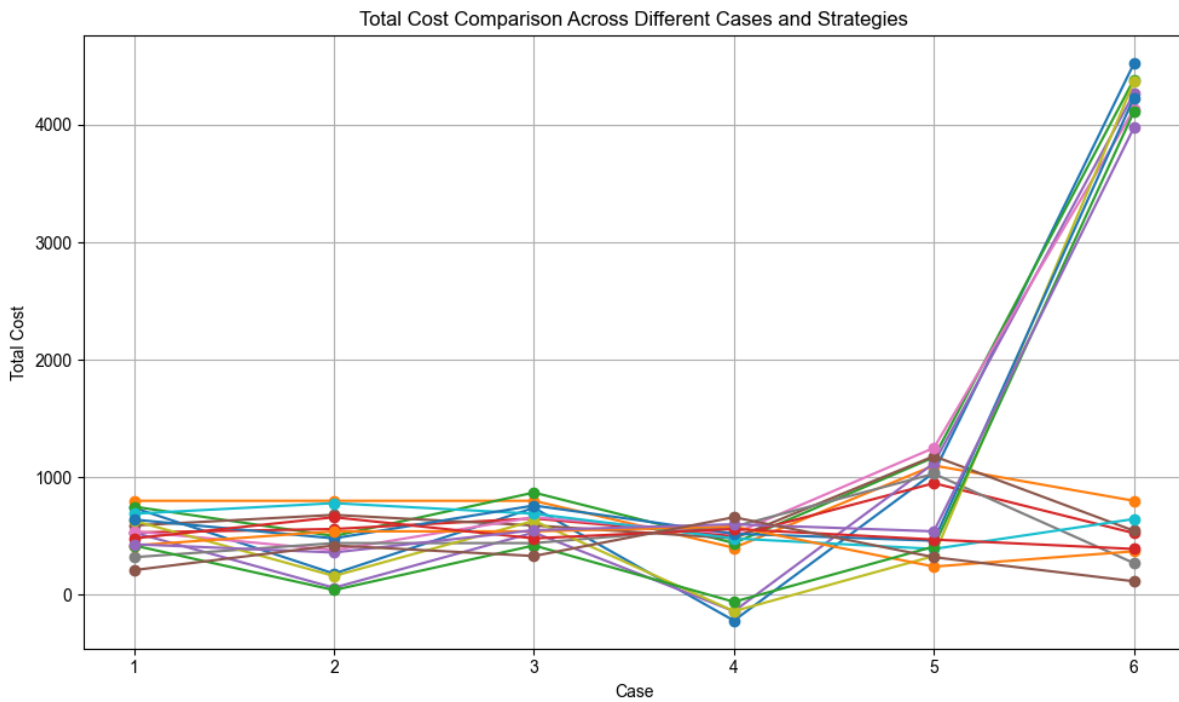


Figure 2. Total Cost Comparison Across Different Cases and Strategies

(3). Decision Support:

Visual analysis enabled companies to intuitively observe cost variations under different strategies, providing robust data support for production decision-making.

4. Conclusion

This research explored the optimization of multi-stage inspection strategies and dynamic defect rate adjustment models in the context of global value chain restructuring and Industry 4.0. By integrating intelligent algorithms and big data analysis, this research aimed to address the challenges of cost reduction and quality control in complex production environments. The following are the key findings and conclusions derived from our research:

(1). Effectiveness of Sampling Inspection:

The introduction of sampling inspection methods significantly improved the accuracy of defect rate estimation. By leveraging hypothesis testing and confidence interval estimation, this research was able to dynamically adjust defect rates at each production stage. This approach not only reduced unnecessary inspection costs but also maintained high levels of quality control, as evidenced by the cost optimization results in different strategy combinations.

(2). Optimal Strategy Identification:

Through comprehensive enumeration and cost analysis, Strategy Combination 8 emerged as the most cost-effective solution, achieving the lowest total cost of approximately 145 yuan at an adjusted defect rate of 2.25%. This strategy balanced inspection precision and cost efficiency, demonstrating its superiority in optimizing production decisions.

(3). Cost Reduction Impact: The implementation of sampling inspection led to a noticeable reduction in total costs across all strategy combinations. This finding underscores the method's effectiveness in minimizing inspection-related expenses while preserving product quality, thereby enhancing operational efficiency and competitiveness.

(4). Practical Implications:

The proposed methodology provides manufacturing enterprises with a robust framework for decision-making in dynamic production environments. By adopting intelligent algorithms and statistical models, companies can better navigate the complexities of global markets, stabilize supply chains, and protect brand reputation.

(5). Future Research Directions:

While this study focused on optimizing inspection strategies within specific production contexts, further research could explore the applicability of these models across diverse industries and production scales. Additionally, the integration of real-time data analytics and machine learning techniques holds promise for further enhancing the precision and adaptability of defect rate control mechanisms.

In summary, this research contributes to the body of knowledge on production efficiency and quality control by offering a data-driven approach to optimizing multi-stage inspection strategies. The findings are particularly relevant in today's era of digital transformation and global competition, where enterprises must continuously innovate to sustain their competitive edge.

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