

Optimization Study of Multibeam Bathymetric Survey Line Based on Multiple Geometric Modeling

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Abstract. This paper focuses on multibeam bathymetric technology in the field of oceanographic survey and seabed mapping, and carries out a systematic study on the optimisation of survey line laying scheme. The purpose of this study is to improve the efficiency and accuracy of oceanographic surveys while reducing resource consumption and survey costs, and to reveal the importance of optimizing the survey line layout by analyzing the influence of the rationality of the survey line layout on the completeness and accuracy of the bathymetric data. Firstly, the importance of optimising the survey line layout is revealed by analysing the influence of the rationality of the survey line layout on the integrity and accuracy of the bathymetric data. On this basis, a series of models are constructed in this paper, including the planar coverage width calculation model, the adjacent strip overlap rate model and the slope coverage width model, which provide quantitative analysis tools for the survey line laying. In the case study of rectangular sea area and real sea area, key indicators such as the total length of the survey line, the percentage of the missed area and the total length of the part with overlap rate more than 20% are calculated to verify the effectiveness of the proposed method. The results show that the optimised line laying scheme can significantly improve the measurement efficiency and reduce the waste of resources, and provides important theoretical support and technical guidance for the practical operation of multibeam survey vessels.

Keywords: Trigonometric function; Trigonometric relationship; Geometric model; Multibeam bathymetry; Optimization of survey line placement.

1. Introduction

In the field of marine survey and seabed mapping, multibeam bathymetric technology is a key means of obtaining high-resolution seabed topographic data. As the complexity of the survey task and accuracy requirements increase, the design of a reasonable survey line deployment scheme has become a key issue in multibeam bathymetry. Many scholars have carried out in-depth research on this issue and proposed a variety of optimization methods. However, existing research has certain limitations. For example, while Li Huijun et al. (2024) pointed out that the reasonableness of survey line deployment directly affects the integrity and accuracy of bathymetric data [1], their research mainly focuses on the theoretical aspect and lacks specific application cases. Deng Zhanyin et al. (2025) emphasized the unique advantages of side-scan sonar multibeam bathymetry technology in specific application scenarios [2], but their research is limited to specific scenarios and lacks general applicability. Kaihua Zhang et al. (2025) highlighted the value of high-precision multibeam technology in bridge sinkhole base trench sweeping tasks [3], but their research is mainly focused on the application of the technology in bridge engineering and lacks exploration in other fields.

In terms of survey line laying optimization, Huang Lijun et al. (2025) proposed a multibeam bathymetric survey line design method based on multi-objective planning and K-means clustering, which effectively improved the efficiency and accuracy of survey line laying [4]. However, this method has certain limitations in dealing with complex terrain. Ge Pingru et al. (2024) constructed a multibeam line model based on computational geometry, which provides new technical support for line laying [5]. Nevertheless, this model is mainly applicable to relatively flat terrain and lacks



adaptability to complex terrain. Xia Hao et al. (2024) simulated and optimized the multibeam survey line, which further enhanced the scientific nature of survey line placement [6]. However, their research mainly focuses on simulation and lacks practical application cases. Zhang Zeyu et al. (2025) developed a multibeam survey line placement model based on multi-objective planning, which can effectively regulate the overlap rate of neighboring survey lines [7]. However, this model has certain limitations in optimizing the total length of the survey line. Meanwhile, the total length of the survey line also affects the execution efficiency and cost of the measurement task. Di Zhi et al. (2024) proposed a multibeam line placement method based on area division, which can effectively shorten the total length of the measurement line while ensuring the coverage requirements [8]. However, this method may lead to uneven distribution of survey lines and affect the accuracy of the measurement results.

The use of multibeam bathymetry in different application scenarios has also attracted much attention. The study of Yi Yuanqin et al. (2025) showed that the multibeam bathymetric system has significant advantages in marine channel surveying [9]. However, their research is mainly focused on the application of the system in marine channels and lacks exploration in other fields. Li Suhao et al. (2024) proposed a route planning model based on multibeam bathymetry, which provides a new idea for practical surveying tasks [10]. Nevertheless, this model is mainly applicable to specific surveying tasks and lacks general applicability. The research of Wenbin Xu et al. (2024) provided a new perspective for solving the sea area line measurement problem in multibeam sounding systems [11]. However, their research mainly focuses on the problem-solving approach and lacks specific application cases. Liang Shennan et al. (2024) explored a multibeam seafloor bathymetry method based on geometric modeling, which further improved the accuracy of the bathymetric data [12]. However, this method has certain limitations in dealing with complex terrain. Li Yuhang et al. (2025) pointed out that the sailing speed has a certain influence on the multibeam bathymetry accuracy of unmanned vessels [13]. However, their research mainly focuses on the influence of sailing speed and lacks exploration of other factors affecting measurement accuracy.

In summary, while multibeam bathymetric technology plays an important role in the field of oceanographic surveying and mapping, the existing research on survey line laying schemes still has certain shortcomings. Some methods lack general applicability, while others have limitations in dealing with complex terrain or optimizing multiple objectives. Therefore, further research is needed to address these issues. In this paper, we aim to explore the optimal design method of multibeam survey line laying by constructing more comprehensive and adaptive models, taking into account the complexity of terrain and multiple optimization objectives. Our goal is to provide more effective theoretical support and technical guidance for the efficient operation of multibeam survey vessels, thereby improving the measurement efficiency and data quality of multibeam bathymetry technology.

Multibeam Line Modeling.

1.1. Plane coverage width calculation model

The working principle of the multibeam bathymetric system is to use the transmitting transducer array to transmit sound waves with wide sector coverage to the seafloor, and through the orthogonality of the pointing of the transmitting and receiving sectors to form the irradiation footprints of the seafloor topography, and appropriate processing of these footprints can reliably depict the three-dimensional characteristics of the seafloor topography. The relevant angular relationship can be solved based on the schematic diagram of the working principle in Figure 1 with specific point information.

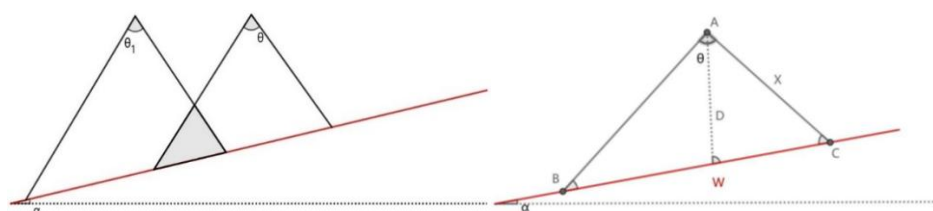


Figure 1. Working principle diagram with specific points

In triangle ΔAWC , let the side AC be x , using the sine theorem with the sum of the interior angles of the triangle as 180° , the following trigonometric equation can be established and solved for x , in triangle ΔABC using the angle relationship with the sine theorem trigonometric equation can be established as follows about the width of the coverage BC :

$$\frac{x_i}{\sin(90^\circ - \alpha)} = \frac{D_i}{\sin\left(90^\circ - \frac{\theta}{2} + \alpha\right)} \quad (1)$$

$$x_i = \frac{D_i \cdot \sin(90^\circ - \alpha)}{\sin\left(90^\circ - \frac{\theta}{2} + \alpha\right)} \quad (2)$$

$$\frac{BC_i}{\sin \theta} = \frac{x_i}{\sin\left(90^\circ - \frac{\theta}{2} - \alpha\right)} \quad (3)$$

$$W_i = \frac{x_i \cdot \sin \theta \cdot \cos \alpha}{\sin\left(90^\circ - \frac{\theta}{2} - \alpha\right)} \quad (4)$$

The above derivation can be used to establish a solved relational equation for the depth of seawater cover:

$$\begin{cases} W_i = \frac{\sin \theta \cdot D_i \cdot \sin(90^\circ - \alpha) \cdot \cos \alpha}{\sin\left(90^\circ - \frac{\theta}{2} - \alpha\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \alpha\right)} \\ D_i = 70 - L_i \cdot \tan \alpha \end{cases} \quad (5)$$

1.2. Modelling of overlap rates between neighbouring bands in the plane

The formula for the overlap rate between adjacent strips is a simplified formula under the condition of flat ground, so according to the definition of the overlap rate as the projection of the slope surface on the horizontal plane, calculate the ratio of the projection of the overlapping part of the length of the horizontal plane to the projection of the length of the previous strip, i.e. the overlap rate with the previous strip, and calculate the overlap rate calculation formula (6), and establish the following relational equation:

$$\eta_i = \frac{(w_2^{i-1} + w_1^i) \cos \alpha - d_i}{(w_1^i + w_2^i) \cdot \cos \alpha} \quad (6)$$

Where w_1^i is the length of the first half of the i th strip, w_2^i is the length of the second half of the i th strip, d_i is the spacing between the i and $i-1$ lines, and if $\eta_i < 0$, it is a missed measurement.

There should be an overlap of 10%~20% between neighbouring strips. η_i is the overlap between the i strip and the previous strip.

1.3. Slope cover width modelling

Establish a three-dimensional right-angle coordinate system with O as the origin, the projection of the slope shoulder on the horizontal plane as the x axis, and the intersection line between the slope surface and the horizontal plane as the y axis, and make a vertical upward straight line perpendicular to the x and y axes, and establish a three-dimensional right-angle coordinate system with it as the z axis as shown in Figure 2.

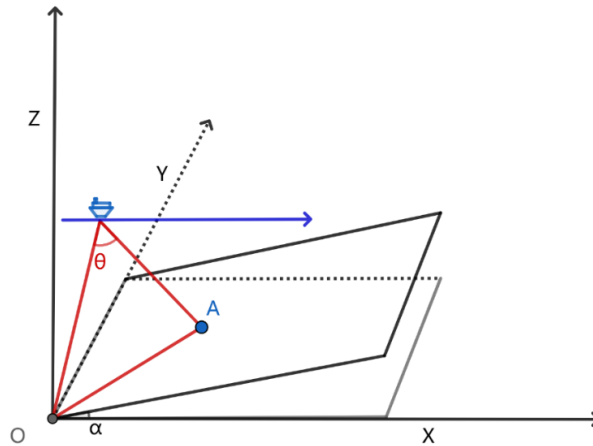


Figure 2. Establishment of coordinate system

According to Figure 2, using the three-dimensional right-angle coordinate system, the corresponding angle and side length values are calculated, and firstly, the relationship equation of the height equation from each point of the slope A to the ground is established:

$$z = x \tan \alpha \quad (7)$$

The top surface of the slope is analysed to establish a relationship between the equations about the height of the points on the top surface of the slope (denoted as A), the projection of the bottom surface of A (denoted as A'), and the projections to the x -axis are made to the apex of A' to intersect with the point of B , and the construction of the triangle $\Delta A'OB$ is shown in Figure 3.

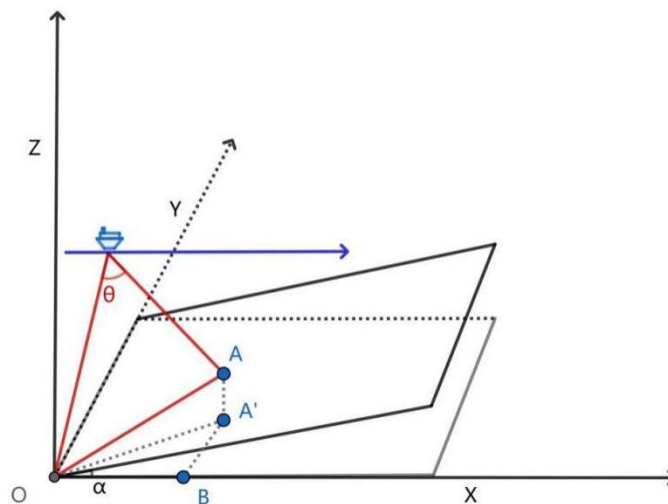


Figure 3. Coordinate diagram after creating auxiliary lines

In the triangle $\Delta A'OB$, the following equation is established based on the relationship between the sum of the interior angles of the triangle being 180° and the angle

$$\angle A'OB = 180^\circ - \beta \quad (8)$$

$$A'O = \frac{b}{\cos(180^\circ - \beta)} \quad (9)$$

In $\Delta A'OA$, let $\angle A'OA = \gamma$, based on the relationship between angles and sides inside a triangle, have:

$$A'A = b \tan \alpha \quad (10)$$

$$\tan \gamma = \frac{A'A}{A'O} = \frac{b \tan \alpha}{\frac{b}{\cos(180^\circ - \beta)}} = \tan \alpha \cdot \cos(180^\circ - \beta) \quad (11)$$

1.4. Modelling of overlap between neighbouring bands

Based on the analysis of the above triangular relationship to solve the coverage width W is given as:

$$\left\{ \begin{array}{l} W = \frac{\sin \theta \cdot D' \cdot \sin(90^\circ - \gamma) \cdot \cos \gamma}{\sin\left(90^\circ - \frac{\theta}{2} - \gamma\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \gamma\right)} \\ D' = 120 - l \cdot \tan \gamma \\ \gamma = \arctan \left[\tan \alpha \cdot \cos(180^\circ - \beta) \right] \end{array} \right. \quad (12)$$

Where D is the depth of the sea at different distances from the centre of the sea, and l is the distance of the measuring vessel from the centre of the sea.

Above is the creation of a geometric model for solving the multibeam bathymetric coverage width.

1.5. Optimisation model for the placement of survey lines in rectangular sea areas

Assume a rectangular sea area with a north-south length of 2 nautical miles and an east-west width of 4 nautical miles, with a depth of 110 m at the centre of the sea area, a depth of 1.5 degrees in the west and a shallowness of 1.5 degrees in the east, and a multibeam transducer with an opening angle of 120 degrees. A set of survey lines is designed with the shortest measurement length that can completely cover the entire sea area to be surveyed, and the overlap rate between neighbouring strips meets the requirement of between 10 and 20 per cent.

A right-angled coordinate system is established with the east-west direction as the x axis and the north-south direction as the y axis. After consulting relevant information, it is known that the seabed coverage can be increased to a greater extent when the direction of the survey line is determined to be parallel to the isobath, so $0^\circ < \beta \leq 90^\circ$. Solve the equation about the coverage width W based on the above model:

$$W = \frac{\sin \theta \cdot D \cdot \sin(90^\circ - \gamma) \cdot \cos \gamma}{\sin\left(90^\circ - \frac{\theta}{2} - \gamma\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \gamma\right)} \quad (13)$$

Record D as the depth of the centre point $110m$. When the two adjacent lines are furthest apart, the total length of the line is minimum, so set the overlap rate of the two adjacent lines as 10%, that is, the distance between the lines is $0.9W$. Determine the coordinates of the starting position as $(0,0)$, take $(0,0)$ as the starting point, and the angle with the x axis as β to establish the first line. Make the next line with the first line spacing $0.9W_i$, and then adjust its angle, rotate the appropriate angle with the centre point as the axis so that it is parallel to the previous line. Where W_i is the coverage width of each point on the first line. When $0^\circ < \beta \leq 90^\circ$, the vertical line from $(0,3704)$ to the first line of measurement and its intersection at H , W' is the first line of measurement on the H point coverage width, set n for the total number of lines, so the objective function is:

$$\begin{cases} n = \frac{7408 \sin \beta}{0.9w} \\ \min l = \frac{2 \cdot 1852}{\sin \beta} n \end{cases} \quad (14)$$

Since this objective function needs to satisfy the condition that the survey line completely covers the entire sea area, there is:

$$\frac{2 \cdot 1852 \sin \beta}{\tan \beta} \leq \frac{W'}{2} \quad (15)$$

Using Lingo analysis and calculation, at $\beta = 90^\circ$, the total length of the line is the shortest, at this time, the total number of lines is 22, the total length of 88,896 metres, that is, 48 nautical miles, the ship sailing line as shown in Figure 4.

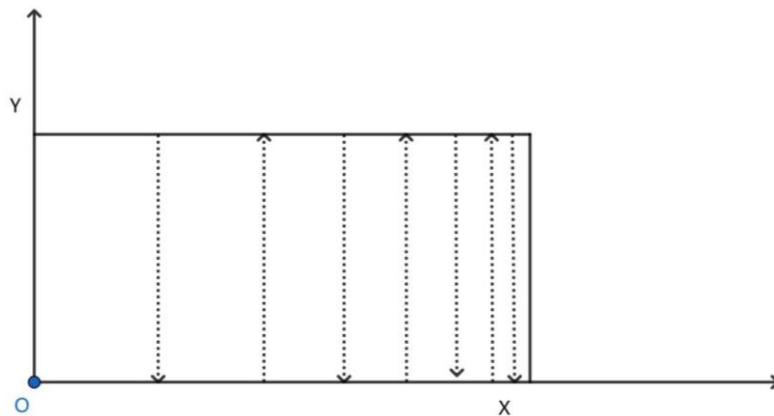


Figure 4. Schematic diagram of the ship's shortest survey line

1.6. Optimisation model for line placement in real waters

Depth data from a single-beam survey of an area 5 nautical miles from north to south and 4 nautical miles from east to west were obtained many years ago. It is now planned to use this data to provide

auxiliary support for the measurement line of the multibeam survey vessel. In designing the survey line, the following requirements need to be met: first, the strips formed by scanning along the line should cover the entire sea area to be surveyed as comprehensively as possible; second, the overlap rate between neighbouring strips should be controlled within 20 per cent as far as possible; and third, the total length of the survey line should be as short as possible. After completing the design of the specific survey line, the following indicators need to be calculated: (1) the total length of the surveyed line; (2) the percentage of the omitted sea area to the total sea area to be surveyed; and (3) the total length of the part of the overlapping area with an overlap rate of more than 20 per cent.

Terrain analysis: Historical data were used to carry out seabed topography analysis to analyse the main features of the seabed (e.g. slope, irregular terrain, etc.) in order to provide directional guidance for the multibeam survey. According to the different depths of seawater in different areas, the sea area was divided into four parts for subsequent calculations.

Model the total length of the part with more than 20 per cent overlap, and let the overlap area be Q , and if there is overlap between the survey lines then

$$Q = W - l \quad (16)$$

The overlap rate is calculated relationally as:

$$\lambda = \frac{Q}{W} \cdot 100\% \quad (17)$$

Then the percentage of missed sea area in the total sea area to be measured is calculated by the model, and the area of missed sea area is set to be M , so the relationship equation about M is established, in which L is the total length of the minimum survey line, and e is the area of the minimised missed sea area.

$$M = S - e \cdot W \cdot L \quad (18)$$

The total length of the overlap rate over 20% T :

$$T = \sum_{i=1}^n (\lambda_i - 20\%) \cdot W (\lambda_i \geq 20\%) \quad (19)$$

Further subregional calculations:

Part 1: The total length of the survey line is shortest when the direction of the survey line is horizontal to the direction of the isobath, so it is assumed that the direction of the survey line is parallel to the direction of the isobath, set $\beta = 90^\circ$ and the schematic diagram is shown in the figure, and the following relationship is established on the basis of the above model, in which Δh is the vertical distance between the highest point and the lowest point on the slope surface, and s is the horizontal distance between the highest point and the lowest point on the slope surface.

$$\left\{ \begin{array}{l} W = \frac{\sin \theta \cdot D' \cdot \sin(90^\circ - \gamma) \cdot \cos \gamma}{\sin\left(90^\circ - \frac{\theta}{2} - \gamma\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \gamma\right)} \\ n = \frac{0.5 \cdot 1852}{0.9w} \\ L = 3.5n + 0.5 \\ \gamma = \arctan\left[\tan \alpha \cdot \cos(180^\circ - \beta)\right] \\ \tan \alpha = \frac{\Delta h}{s} \end{array} \right. \quad (20)$$

The first part of the solution yields $L = 75515.3m$.

Part 2: This part is analysed to establish a 3D Cartesian Coordinate System, combining Figure 5 with the following relational equation based on the previous model:

$$\left\{ \begin{array}{l} W = \frac{\sin \theta \cdot D' \cdot \sin(90^\circ - \gamma) \cdot \cos \gamma}{\sin\left(90^\circ - \frac{\theta}{2} - \gamma\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \gamma\right)} \\ n = \frac{\sqrt{26.5} \cdot 1852}{0.9w} \\ \tan \alpha = \frac{\Delta h}{s} \\ L = \frac{\sqrt{26.5} \cdot n \cdot 1852}{2} \\ \gamma = \arctan\left[\tan \alpha \cdot \cos(180^\circ - \beta)\right] \end{array} \right. \quad (21)$$

The second part of the solution yields $L = 433785.7847m$.

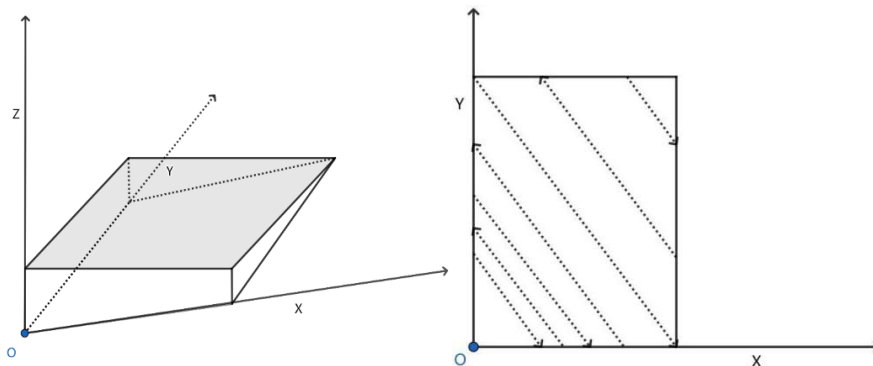


Figure 5. Schematic plan and coordinates of part 2

Part 3: The part is analysed to establish the following relational equation:

$$\left\{ \begin{array}{l} W = \frac{\sin \theta \cdot D' \cdot \sin(90^\circ - \gamma) \cdot \cos \gamma}{\sin\left(90^\circ - \frac{\theta}{2} - \gamma\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \gamma\right)} \\ n = \frac{\sqrt{1.5^2 + 4.5^2} \cdot 1852}{0.9w} \\ L = \frac{\sqrt{1.5^2 + 4.5^2} \cdot n \cdot 1852}{2} \\ \gamma = \arctan\left[\tan \alpha \cdot \cos(180^\circ - \beta)\right] \\ \tan \alpha = \frac{\Delta h}{s} \end{array} \right. \quad (22)$$

The third part of the solution yields $L = 146323.4372m$.

Part 4: The part is analysed to establish a three-dimensional Cartesian coordinate system, and the following relationships are established on the basis of the previous model:

$$\left\{ \begin{array}{l} W = \frac{\sin \theta \cdot D' \cdot \sin(90^\circ - \gamma) \cdot \cos \gamma}{\sin\left(90^\circ - \frac{\theta}{2} - \gamma\right) \cdot \sin\left(90^\circ - \frac{\theta}{2} + \gamma\right)} \\ n = \frac{0.5 \cdot \sqrt{2} \cdot 1852}{0.9w} \\ L = \frac{0.5 \sqrt{2} \cdot 1852 \cdot n}{2} \\ \gamma = \arctan\left[\tan \alpha \cdot \cos(180^\circ - \beta)\right] \\ \tan \alpha = \frac{\Delta h}{s} \end{array} \right. \quad (23)$$

The fourth part solution yields $L = 606.9638m$.

This indicates that when the survey line direction is perpendicular to the isobath direction, it can minimize the duplicate coverage of the survey line in the horizontal direction and improve the efficiency of the measurement, and it is also in line with the common principle of survey line placement in marine surveying, i.e., the direction of the survey line should be the same as or perpendicular to the seafloor isobath direction as much as possible in order to obtain more accurate seafloor topographic information.

2. Results

2.1. Optimisation model solution for the placement of survey lines in rectangular sea areas

Using Lingo analysis and calculation, at $\beta = 90^\circ$, the total length of the line is the shortest, at this time, the total number of lines is 22, the total length of 88,896 metres, that is, 48 nautical miles, the ship sailing line as shown in Figure 6.

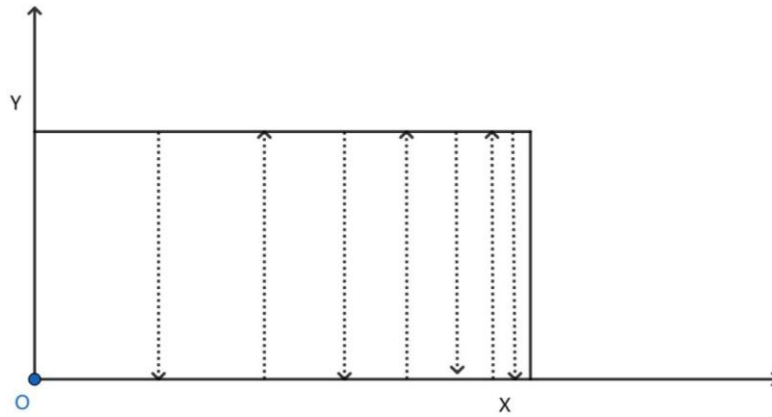


Figure 6. Schematic diagram of the ship's shortest survey line

This indicates that when the survey line direction is perpendicular to the isobath direction, it can minimize the duplicate coverage of the survey line in the horizontal direction and improve the efficiency of the measurement, and it is also in line with the common principle of survey line placement in marine surveying, i.e., the direction of the survey line should be the same as or perpendicular to the seafloor isobath direction as much as possible in order to obtain more accurate seafloor topographic information.

2.2. Optimisation model solution for real sea line placement

Due to the complex topography of the real sea area, it is divided into four parts to be solved separately. The above model and optimization method are applied to each part to obtain the results such as the length of the survey line in each area, and the results are as follows:

The first part of the solution yields $L = 75515.3m$.

The second part of the solution yields $L = 433785.7847m$.

The third part of the solution yields $L = 146323.4372m$.

The Part IV solution yields $L = 606.9638m$.

In terms of the length of the survey line in each part, the length of the survey line in the first part is 75,515.3 meters, in the second part is 433,785.7847 meters, in the third part is 146,323.4372 meters, and in the fourth part is 606,963.8 meters. The large difference in the length of the survey line in different parts is mainly due to the complex and variable topography of the real sea area, with different seawater depths, slopes and other factors in each area, which leads to different difficulties and requirements for the deployment of the survey line.

The total survey line length is shortest when the direction of the first part of the line is parallel to the direction of the isobath, which is in line with the commonly used principle of laying survey lines in oceanographic surveying, that is, to follow the characteristics of the terrain, so as to make better use of the coverage characteristics of the multibeam bathymetric system, to reduce the meandering and repetition of the survey line, and to increase the efficiency of the survey. The second, third and fourth parts of the survey line laying need to comprehensively consider the three-dimensional terrain factors, and through the establishment of a three-dimensional right-angle coordinate system and the corresponding geometric model, the direction and spacing of the survey line are accurately calculated to adapt to the changes in the complex terrain and to ensure the accuracy and completeness of the measurement.

The results of the four parts were integrated to produce the final results of the total length of the survey line for the whole sea area, the percentage of the area of the missed sea area and the total length of the part with an overlap rate of more than 20%, as follows:

The total length of the line over the entire sea area is $L = 656231.4857m$.

Percentage of missed sea area to total sea area to be measured 44.1424 per cent.

The total length of the portion of the overlap area with more than 20 per cent overlap is 44157.4443m.

The total length of the survey line is 656,231.4857 m. This length is optimized under the conditions of satisfying the full coverage of the survey line and controlling the overlap rate, etc. Compared with the unoptimized scheme, the survey distance may be greatly shortened, the survey workload and resource consumption are reduced, and the efficiency of the survey is improved, which embodies the value of the optimization model in the application of the complex sea area.

3. Conclusion

In this paper, by constructing the planar coverage width calculation model, the adjacent strip overlap rate model and the slope coverage width model, the proposed optimisation strategy can significantly improve the efficiency and accuracy of the survey line laying. The optimised line placement scheme effectively reduces the total length of the line while ensuring full coverage, controls the overlap rate of adjacent strips and minimises the missed area.

In a rectangular sea area with a length of 2 nautical miles from north to south and a width of 4 nautical miles from east to west, the total length of the optimized survey line is 48 nautical miles, which completely covers the entire area to be surveyed, and the overlap rate of adjacent strips is controlled to be between 10 per cent and 20 per cent, thus verifying the effectiveness and feasibility of the proposed method. In the real sea area with a length of 5 nautical miles from north to south and a width of 4 nautical miles from east to west, despite the complex topography of deep in the west and shallow in the east, the optimized line laying scheme significantly reduces the total length of the line, and the percentage of the missed area is reduced to 44.1424 per cent, and the total length of the part with overlap rate of more than 20 per cent is 44,157.4443 metres. This result shows that the proposed method can effectively cope with the complex terrain, and provides reliable theoretical support and technical guidance for the actual measurement tasks. The research results in this paper not only improve the measurement efficiency and data quality of multibeam bathymetry technology, but also provide a scientific basis for the actual operation of multibeam survey ships.

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