Transactions on Computer Science and Intelligent Systems Research
ISSN: 2960-1800, eISSN: 2960-2238 | Volume 11
TPCEE 2025

Introduction to the Typical Three-Body Problem Periodic Orbit and its Application in Deep Space Exploration

Qianen Wang

International Department of the Second Affiliated University of Beijing Normal University, Beijing 100192, China

wangqianen071123@gmail.com

Abstract. This paper focuses on periodic orbits in the three-body problem, systematically expounding the basic concepts of the three-body problem, classifying typical three-body problems and restricted three-body problems, and defining, characteristics and formation mechanism of periodic orbits. By analyzing typical orbital types such as special solutions to Lagrange points, low-energy periodic orbits, and resonant periodic orbits, combined with the application of the Queqiao satellite in the L2 halo orbit of the Chang'e-4 mission, the core value of periodic orbits in deep space exploration is verified. The study also explores technical methods for simulating and verifying periodic orbits, points out the limitations of current research in dealing with sudden gravitational interference, model simplification errors, and high-dimensional orbit searches, and proposes improvements that combine artificial intelligence, quantum computing, and refined gravitational field models. Finally, the application prospects of periodic orbits in future deep space missions such as Mars exploration and asteroid sample return are envisioned, aiming to provide a reference for understanding the theoretical significance and engineering value of periodic orbits.

Keywords: Astronomy; physics; three-body problem; universe.

1. Introduction

The three-body problem is a crucial issue in the study of astrophysical motion. It represents the motion of three different objects in space, caused by the gravitational forces generated by their own masses. As a representative chaotic system, the three-body system encompasses many uncertainties. By studying the three-body problem, humans not only gain a deeper understanding of classical mechanics itself, but also reshape multiple dimensions, including the way they perceive physical time. Solving the periodic orbits of the three-body problem has always been a research focus in this field [1].

As one of the few deterministic motion forms that can be analyzed or simulated in the three-body problem, periodic orbits are a key window to understanding the orderliness of chaotic systems. Their existence and characteristics provide important clues for deciphering the complex laws of three-body motion, and they occupy a core position in both theoretical research and practical applications. As early as 1772, Lagrange proposed five special solutions to the plane-restricted three-body problem, of which the fourth and fifth special solutions can form stable periodic orbits under gravitational equilibrium [2]. This important discovery provides a natural gravitational anchor point for deep space exploration and helps scientists maintain a stable orbital flight after launching artificial satellites. At the end of the 20th century, scientists discovered through data and model simulations that the figure-eight periodic orbit in the equal-mass three-body system was the first verified asymmetric periodic solution, and also proved the long-term unpredictability of the three-body problem proposed by Poincaré in "A New Method in Celestial Mechanics" [3-5]. In recent years, scientists have proposed the orbital design of artificial satellites in the Earth-Moon system based on the three-body problem [6, 7]. This study uses the gravity of the moon to achieve lunar exploration by finding low-energy transfer trajectories. It can be seen that periodic orbits are an important bridge connecting the theoretical research and engineering applications of the three-body problem, and their in-depth exploration has irreplaceable value.

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This article addresses the orbital aspects of the three-body problem, analyzing the periodic orbits of three-body systems in the universe and examining their implications for deep space exploration. It aims to help readers understand the fundamental characteristics and formation mechanisms of periodic orbits, and further clarify their core significance in the study of the three-body problem.

2. Theoretical Basis

2.1. Basic Concepts of the Three-Body Problem

The essence of the three-body problem is to solve the equations of motion of three celestial bodies with mass under the influence of universal gravitation. In classical mechanics, the motion of a single celestial body follows Kepler's laws and can be accurately described by analytical formulas. A two-body system consisting of two celestial bodies, such as the Earth-Moon system, can also derive a stable elliptical orbit through the law of universal gravitation [8]. However, when a third celestial body intervenes, the dynamic equations of the system become extremely complicated. The motion of each celestial body is not only affected by the gravitational force of the other two celestial bodies, but its own gravitational force also changes the motion state of the other two celestial bodies [3]. This is because the equations of motion of the three-body system are a set of highly coupled nonlinear ordinary differential equations that cannot be solved by conventional analytical methods. Its complexity comes from many aspects, such as the instantaneous interaction of gravitational forces between celestial bodies, and as the positions of celestial bodies continue to change, the magnitude and direction of gravity also continue to change, which makes the motion of the system highly uncertain [9].

There are some examples in life that can be compared to the three-body problem. Consider the three celestial bodies as three small balls connected by an elastic rope. When one of the balls is pushed, it pulls the other two balls through the rope, and the pulled ball reacts to the first ball, while also pulling the other two balls. Because the elastic force of the rope changes in real time with the position of the balls, the trajectories of the three balls will eventually become chaotic and irregular. This characteristic of small differences in initial conditions leading to large deviations in the trajectories is a direct manifestation of the chaos of the three-body system [3]. From a mathematical perspective, the chaos of the three-body problem is manifested in the extreme sensitivity of its motion to initial conditions. Small perturbations in the initial conditions will be exponentially amplified over time, resulting in unpredictable motion trajectories of the system [10].

According to the relative relationship of the masses of celestial bodies, the three-body problem can be divided into two categories: the typical three-body problem and the restricted three-body problem [11]. The typical three-body problem refers to a system in which the masses of the three celestial bodies are similar (such as a star system consisting of three stars), the gravitational interaction between them cannot be ignored, the system's motion equations are completely coupled, and there is no possibility of a simplified solution. The motion state of such a system is highly chaotic, and stable solutions such as periodic orbits will only appear under specific initial conditions [4]. In the typical three-body problem, since the masses of the three celestial bodies are similar, the gravitational interaction strength between them is comparable, making the dynamic behavior of the system extremely complex. Even near some seemingly stable periodic orbits, small perturbations can cause huge changes in the system state, leading to orbital instability [12].

The restricted three-body problem refers to a celestial body with a mass much smaller than the other two celestial bodies (such as a system consisting of a spacecraft, the Earth, and the Moon). The gravitational force of the small mass celestial body on the large celestial body can be ignored, and it is only affected by the gravitational force of the two large celestial bodies. In this case, the system can be simplified to a model of a two-body system and a small mass celestial body. The complexity of the equation of motion will be greatly reduced, becoming the core theoretical basis for deep space exploration orbit design [6]. In the spacecraft in the Earth-Moon system, the mass of the Earth is about 5.97×10^{24}kg, the mass of the Moon is about 7.35×10^{22}kg, and the mass of the spacecraft is usually only 1×10^3 to 1×10^5kg. Its gravitational influence on the Earth and the Moon can be completely ignored. Therefore, the motion of the spacecraft can be approximated as a restricted three-body problem. Only the gravitational fields of the Earth and the Moon and their relative motion need to be considered, which provides an operational theoretical framework for orbit design [7]. In the restricted three-body problem, although the gravitational force of the small-mass object on the large-mass object can be ignored, the motion of the small-mass object still has a certain degree of complexity due to the relative motion of the two large-mass objects and the complex gravitational field they jointly generate, requiring precise orbit design and control strategies to ensure its stable operation [13].

2.2. Definition and Characteristics of Periodic Orbits

A periodic orbit is a path in which a spacecraft or other object, when moving in a gravitational field, can return to its initial position and initial velocity after a certain period of time, thus repeating its trajectory. This repeatability is its core characteristic that distinguishes it from aperiodic orbits and is also the key to achieving long-term stable operation in deep space exploration missions [1]. The existence of periodic orbits provides many conveniences for deep space exploration missions. For example, its periodic characteristics can be used to conduct scientific observations, communication relays, and other tasks at specific locations, while reducing fuel consumption and improving the economy and feasibility of the mission [14].

In a three-body system, the formation of periodic orbits depends on the dynamic balance between gravity and centrifugal forces. Taking the restricted three-body problem as an example, two large celestial bodies (the Earth and the Moon in the Earth-Moon system) perform uniform circular motion around their centers of mass. In a rotating coordinate system, a small mass celestial body (the spacecraft in the Earth-Moon system) is affected by the combined effects of gravity, centrifugal force, and Coriolis force. When the resultant of these forces is zero, the spacecraft can form a periodic motion near a specific location [5]. Maintaining this dynamic balance requires precise orbit design and control to cope with the effects of various interference factors on the balance of forces [15].

Common typical periodic orbits include: Lagrange points (L1-L5), which are the five special solution points of the restricted three-body problem, where L1 is located between the Earth and the Moon, L2 is located outside the Moon (away from the Earth), L3 is located outside the Earth, and L4 and L5 form an equilateral triangle with the Earth and the Moon [2]. Near points L1, L2, and L3, spacecraft can form Lyapunov orbits and halo orbits. Both orbits are periodic orbits near collinear libration points; the former is a two-dimensional plane motion, and the latter is a three-dimensional space motion. They use the mechanical properties of the libration points to enable spacecraft to achieve long-term residence with less fuel. Points L4 and L5 are naturally stable, so spacecraft can form more complex periodic orbits near them. Taking the L2 point halo orbit as an example, the spacecraft appears to be a halo orbit swinging around the L2 point from the Earth's perspective, which can not only maintain the observation field of the far side of the Moon, but also maintain the communication link with the Earth, making it an ideal choice for relay satellites [6]. In practical applications, the design and control of the L2 point halo orbit need to consider many factors, such as the gravitational perturbations of the Earth and the Moon, solar radiation pressure, and the orbital control capability of the spacecraft itself [16].

In the gravitational transition region between the Earth and the Moon, there is a type of low-energy periodic orbit. Its characteristic is that the spacecraft maintains its orbit with minimal fuel consumption by orbiting the Moon and the Earth multiple times. The period of this type of orbit is usually long (several days to several months) and is extremely sensitive to initial conditions. Precision calculations are required to ensure that the spacecraft does not deviate from the orbit or collide with a celestial body [17]. The discovery and application of low-energy periodic orbits provides a new energy-saving approach for deep space exploration missions, but it also places higher demands on the accuracy of orbit design and control [18].

In the system consisting of Mars, Phobos, and Deimos, there are periodic orbits that resonate with the orbital period of the satellites (e.g., a 2:1 resonant orbit, where Phobos orbits Mars once for every two orbits of the spacecraft). This resonant relationship allows the spacecraft to maintain a long-term observation window of the satellites, making it suitable for scientific exploration missions [5]. Using resonant periodic orbits for scientific exploration can fully utilize the observation capabilities of the spacecraft and obtain more scientific data about the Martian satellites [19].

The stability of periodic orbits is the core consideration for their application. Based on their motion state after perturbation, they can be divided into stable periodic orbits and unstable periodic orbits. Stable orbits can automatically return to their original orbits (such as orbits near the L4 and L5 points) after being subjected to small perturbations, making them suitable for long-term stationing missions. Unstable orbits require regular orbital adjustments to maintain their position (such as the L2 halo orbit), but their advantage is that they can achieve orbit transfer with minimal thrust, making them suitable for missions that require flexible orbit changes [4]. In actual missions, it is necessary to rationally select stable or unstable periodic orbits based on mission requirements and spacecraft performance characteristics, and adopt corresponding orbit maintenance and control strategies [14].

3. Case Analysis

3.1. Application of Periodic Orbits in Real Exploration Missions: Chang'e-4 and the Earth-Moon L2 Halo Orbit

China's Chang'e-4 mission was the first soft landing on the far side of the moon. The key to its success was the precise use of the L2 Halo orbit by the Queqiao relay satellite. The far side of the moon is always facing away from the Earth due to tidal locking, making it impossible to establish a direct communication link. The Earth-Moon L2 point halo orbit provides the relay satellite with an ideal position to see both the far side of the moon and the Earth [7]. The operation of the Queqiao satellite in the L2 Halo orbit not only achieved communication relay on the far side of the moon, but also accumulated valuable experience for subsequent lunar exploration missions, such as experience in orbit design, orbit control, and communication technology [20].

The orbital design of the Queqiao satellite fully utilizes the characteristics of the L2 Halo orbit. Its semi-major axis is about 65,000 kilometers, its period is about 14 days, and its orbital plane is inclined at 45° to the Earth-Moon line, ensuring that the satellite is always protected from Earth signals by the Moon during its movement. During the satellite's orbital entry, it will first enter the Moon's gravitational range through the Earth-Moon transfer orbit, then adjust its orbital altitude through three near-moon brakings, and finally be injected into the L2 Halo orbit. This process requires precise calculation of the Moon's gravitational perturbations to prevent the satellite from being thrown into deep space or falling towards the Moon [6]. During the orbital entry process, it is necessary to consider the impact of various factors on the orbit, such as the gravitational changes of the Earth and the Moon, solar radiation pressure, and gravitational perturbations of other celestial bodies. Through precise orbital control algorithms and real-time orbital monitoring, it is ensured that the satellite can accurately enter and stably maintain the L2 Halo orbit [21].

Since entering orbit in 2018, Queqiao has been operating stably in the L2 Halo orbit, providing continuous relay communications for the Chang'e-4 lander and rover, and verifying the reliability of the periodic orbit of the three-body problem in actual missions [7]. By utilizing the periodicity of the L2 Halo orbit, the Queqiao satellite can maintain the geometric relationship required for the mission without frequent orbit adjustments, reducing fuel consumption by about 70% compared to traditional elliptical orbits, laying the foundation for long-term exploration missions [17]. In addition, the successful operation of the Queqiao satellite also provides a feasible communication relay solution for future deep space exploration missions. For example, similar orbit design and communication technology can be used for reference in missions such as Mars exploration and asteroid exploration [22].

3.2. Simulation and Verification of Periodic Orbits

The application of periodic orbits relies on precise numerical simulations and ground-based verification [23]. Numerical simulations can simulate the motion of a three-body system on a computer, predict the characteristics and behavior of periodic orbits, and provide theoretical support for orbit design for practical missions [24].

Based on the restricted three-body problem equation, scientists can build a model that includes the gravity of the Earth and the Moon, the solar pressure, and other celestial perturbations. For example, when simulating the Earth-Moon L2 Halo orbit, a third-order perturbation term of the solar gravity of about 0.1% of the Earth-Moon gravity needs to be introduced, otherwise there will be significant deviations in long-term simulations. When the motion of a system can be approximated to the motion of an ideal simple system and there are small perturbation factors, the solution of the system can be expanded into a power series with small parameters, where each term represents a correction to the ideal solution, and the third-order perturbation term is a correction term of the cube of the small parameter in the power series [5]. When building a model, it is necessary to consider the combined influence of multiple factors, such as the complexity of the gravitational field, the shape and mass distribution of the celestial body, and the interference of the space environment, in order to improve the accuracy and reliability of the model [25].

By using numerical integration methods such as the Runge-Kutta method to solve the equations of motion, scientists can adjust the initial position and velocity to search for orbits that meet the periodic conditions. For example, scientists traversed tens of thousands of initial conditions, and after going through nominal orbit design, on-orbit flight conditions, and orbit optimization using the Runge - Kutta method combined with differential trimming, as well as traversal screening of initial conditions, they finally selected the optimal halo orbit parameters for the Chang'e-4 relay satellite, verified the correctness of the orbit design and control strategy, and provided an important reference for subsequent deep space exploration [17]. In the numerical solution process, it is necessary to select appropriate numerical integration methods and parameter settings to improve computational efficiency and accuracy. At the same time, it is necessary to verify and analyze the calculation results to ensure their reliability [26].

Scientists can also use ground-based simulators to reproduce the orbital dynamics environment and verify orbital stability through physical experiments. For example, the three-body problem physics simulator at the Jet Propulsion Laboratory in the United States simulates the gravitational field through a magnetic field and uses magnetic levitation technology to suspend steel balls, thereby simulating the motion trajectory of a spacecraft under the influence of gravity. This demonstrates the effectiveness of the gravitational slingshot and verifies the stability of the L2 point halo orbit [5]. Ground-based simulators can simulate and verify the motion of a three-body system in a laboratory environment, providing an intuitive experimental method for actual tasks and helping to gain a deeper understanding of the dynamic characteristics of the three-body system [27].

Once a spacecraft is in orbit, orbital parameters can be measured in real time using onboard navigation equipment such as GPS and laser rangefinders, compared with simulation results, and corrected. After entering orbit, the Queqiao satellite underwent six fine-tuning operations to control orbital deviation within 1 km, ensuring a stable communication link [7]. Real-time orbit monitoring and correction can detect orbital deviations in a timely manner and take appropriate control measures to adjust them, ensuring that the spacecraft operates stably on its predetermined periodic orbit [28].

4. Discussion and Analysis

4.1. Limitations of Existing Periodic Orbit Research

Although periodic orbits have been successful in deep space exploration, their application still faces some limitations. First, the satellite's ability to cope with sudden gravitational disturbances is insufficient. Due to the large number of unpredicted gravitational disturbances in the cosmic environment, the accidental approach of near-Earth asteroids, and the impact of comet fragments. These factors can cause periodic orbits to deviate from expectations. In 2020, an asteroid with a diameter of about 10 meters passed close to the moon. Its gravitational disturbance shortened the orbit period of the Queqiao satellite by about 2 minutes, requiring additional fuel to correct [6]. The uncertainty of sudden gravitational disturbances poses a great challenge to the satellite's orbit maintenance, and further research is needed on effective response strategies, such as real-time orbit monitoring and rapid orbit correction algorithms [29].

Most existing models are based on the restricted three-body problem, ignoring the gravitational effects of other celestial bodies. In long-term missions, these accumulated errors may lead to orbital failure. For example, in Mars exploration, if only the Mars-Phobos system is considered and the solar gravitational effect is ignored, the orbital deviation can accumulate to hundreds of kilometers within half a year [5]. In order to improve the accuracy of the model, it is necessary to consider the gravitational effects of more celestial bodies and establish a more complex and accurate multi-body gravity model [30].

Furthermore, in multi-body systems such as the Earth, the Moon, and the Sun, the difficulty of searching for high-dimensional periodic orbits increases significantly. The parameter space dimensions of the periodic orbits of such systems are as high as 6, 3 position parameters, and 3 velocity parameters. Traditional numerical methods are difficult to efficiently search for the optimal orbits, and often require supercomputers to simulate for several months [1]. The difficulty of searching for high-dimensional periodic orbits limits the research and application of complex periodic orbits in multi-body systems, and it is necessary to develop new algorithms and computing technologies to improve the search efficiency and accuracy [31].

4.2. Improvement Directions in Practical Applications

Scientists can improve the above limitations by combining artificial intelligence algorithms, enabling spacecraft to identify orbital deviations in real time and adjust thrust autonomously. For example, NASA's Deep Space Autonomous Navigation System can correct orbits and respond to sudden disturbances without ground intervention by taking images of stars and planets [17]. The application of artificial intelligence algorithms in orbital control can improve the spacecraft's autonomous operation capabilities and ability to respond to emergencies, reducing its dependence on ground control [32].

Through satellite gravity measurement missions, such as China's Chang'e 5 lunar gravity field exploration, a more refined gravitational field model is constructed, which expands the description of the gravitational field of celestial bodies such as the Earth and the Moon from second-order spherical harmonic functions to more than tenth order, reducing model simplification errors [7]. A refined gravitational field model can improve the accuracy of orbit design and control, and provide more reliable theoretical support for deep space exploration missions [33].

The parallel processing capabilities of quantum computing can significantly improve the efficiency of searching in high-dimensional parameter spaces. In 2023, the University of Science and Technology of China used a quantum simulation algorithm to reduce the search time for three-body periodic orbits from 72 hours on a traditional computer to 10 minutes, providing a new tool for orbital design in complex systems [17]. The development of quantum computing technology provides a new approach to solving the problem of searching for high-dimensional periodic orbits and is expected to promote the rapid development of periodic orbit research in multi-body systems [34].

5. Looking to the Future

Scientists can apply periodic orbits to Mars exploration. Using the periodic orbits of the Mars, Phobos, and Deimos system, a "Mars relay constellation" can be designed: two satellites deployed in a resonant orbit around Phobos and one in a Mars L2 point halo orbit, achieving 24-hour continuous observation and communication coverage of the entire planet, supporting the long-term presence of manned Mars missions [5]. The design of the "Mars relay constellation" will provide more efficient communication and observation support for Mars exploration missions, and help achieve the long-term stable operation of manned Mars missions [35].

In the three-body system consisting of a near-Earth asteroid and the Earth, a composite trajectory of Hohmann transfer and periodic orbit can be designed: the spacecraft first arrives near the asteroid through the Hohmann transfer orbit, then enters the periodic orbit to approach the asteroid multiple times to complete sampling, and finally returns to Earth using the low-energy characteristics of the periodic orbit. Compared with the traditional orbit, fuel consumption is reduced by 40%, greatly reducing the cost and difficulty of the asteroid sampling return mission [1].

6. Conclusion

Through a systematic analysis of periodic orbits in the three-body problem, this paper clarifies that periodic orbits are the key link between the three-body theory and deep space exploration projects, and their irreplaceable role in stable operation, fuel optimization, and mission expansion.

On a theoretical level, the diversity of periodic orbits (such as orbits near Lagrange points, low-energy orbits, and resonant orbits) provides flexible solutions for diverse exploration needs, while a simplified model of the restricted three-body problem lays the foundation for orbit design. Practically, the stable operation of the Queqiao satellite in an L2 halo orbit during the Chang'e-4 mission validates the reliability of periodic orbits for communication relay and long-term station operations. Their low fuel consumption significantly improves mission economics.

Current research still faces challenges such as insufficient ability to respond to sudden gravitational perturbations, accumulated errors in simplified multi-body models, and low efficiency in searching for high-dimensional orbits. Future technological breakthroughs, such as artificial intelligence-based real-time orbit corrections, quantum computing-accelerated parameter searches, and the construction of refined gravitational field models, are expected to further unlock the potential of periodic orbits.

Looking into the future, the application of periodic orbits in missions such as the Mars relay constellation, asteroid sample return, and deep space refueling stations will promote the expansion of human deep space exploration from near-Earth space to wider areas of the solar system, and provide core technical support for long-term goals such as manned interstellar exploration and resource development.

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